# Mathematics

# Grade 7

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# Preface

School education is the foundation for preparing the citizen who are loyal to the nation and nationality, committed to the norms and values of federal democratic republic, self-reliant and respecting the social and cultural diversity. It is also remarkable for developing a good moral character with the practical know-how of the use of ICT along with the application of scientific concept and positive thinking. It is also expected to prepare the citizens who are moral and ethical, disciplined, social and human value sensitive with the consciousness about the environmental conversation and sustainable development. Moreover, it should be helpful for developing the skills for solving the real life problems. This textbook 'Mathematics, Grade 7' is fully aligned with the intent carried out by the National Curriculum Framework for School Education, 2076 and is developed fully in accordance with the new Basic Level Mathematics Curriculum (Grade 6-8), 2077.

This textbook, initially written by Ms. Anupama Sharma, Mr. Dev Narayan Yadav, Mr. Medani Prasad Kadel and Mr. Jagannath Adhikari, has been revised by a team of experts Mr. Lok Nath Bhattarai, Mr. Santosh Shrestha, Mr. Ganesh Prasad Sapkota, Ms. Pramila Bakhati and Mr. Jagannath Adhikari. It has been translated by a team of experts Mr. Naba Raj Pathak, Mr. Daya Raj Acharya, Ms. Pramila Bakhati, Mr. Ram Chandra Dhakal and Mr. Jagannath Adhikari. The contribution made by Director General Ana Prasad Neupane, Prof. Dr. Ramjee Prasad Pandit, Mr. Keshab Raj Phulara, Mr. Ram Hada and Ms. Nirmala Gautam is remarkable in bringing the book in this form. The language of this book was edited by Mr. Nabin Kumar Khadka. The layout was designed by Mr. Nawaraj Puri. The Curriculum Development Centre extends sincere gratitude to all of them.

The textbook is a primary resource for classroom teaching. Considerable efforts have been made to make the book helpful in achieving the expected competencies of the curriculum. Curriculum Development Centre always welcomes constructive feedback for further betterment of its publications.

2079 B.S.

**Curriculum Development Centre**
**Sanothimi, Bhaktapur**

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# Lesson 1 Set

## 1.0 Review

Discuss in pairs and write the following sets in set notation:

(a) The set of seven days of a week

(b) The set of vowel letters in English alphabet

(c) The set of colors used in the national flag of Nepal

(d) The set of square numbers up to 50

(e) The set of prime numbers up to 15

## 1.1 Types of Set

### 1.1.1 Null Set

**Activity 1**

Study the following set. How many elements are there? Discuss and find out the conclusions:

(a) The set of female Prime Minister of Nepal

(b) The set of boy-student studying in Kanya School

(c) The set of natural numbers between 1 and 2

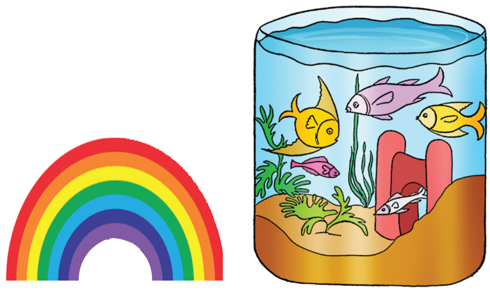
There is no element in the above three set. A set which does not contain any element is called an empty set or null set or void set. It is denoted by the Greek symbol $\emptyset$. $\emptyset$ is read as 'Phi'

A set which does not contain any element is called an empty set or Null set or void set. It is denoted by $\emptyset$ or $\{ \}$.

## 1.1.2 Finite and Infinite Sets

### Activity 2

Observe the following pictures and discuss by constructing the required sets:



(a) How many colors are there in the set of colors in rainbow?

(b) How many fish are there in the set of fish inside the aquarium?

(c) How many numbers are there in the set of natural numbers?

From the above pictures, it can be said with certainty that the colors of the rainbow and the number of fish in the aquarium can be counted. But the number of elements in the set of natural numbers is not finite.

(a) A set is said to be finite set if it contains finite or fixed number of elements.

(b) A set that has no definite number of elements is called infinite set.

### Activity 3

1. Study the following sets and discuss whether they are finite or infinite.

   N = {natural numbers}

   W = {whole numbers less than 30}

   W = {even numbers}

   S = {square numbers up to 30}

(a) What are the elements of the set N, W, E and S?

(b) Is it possible to say that the number of elements of the set N, W, E and S is definitely fixed?

The first element of both Set N and E are known as the first elements but the last elements are not known. It is not possible to say for sure that there are so many elements when the last element is not known. Both sets have infinite elements. So both N and E are infinite sets. The elements of both sets W and S are definite. So, W and S are finite sets.

### Example 1

**State whether the following sets are finite or infinite. Write the number of elements if they are finite.**

(a) The set of whole numbers

(b) The set of factors of 24

#### Solution

Here,

(a) Let W = The set of whole numbers,

W = {0, 1, 2, 3, …}

There are infinite number of elements in the set W. So W is infinite set.

(b) Let F = The set of factors of 24.

F = {1, 2, 3, 4, 6, 8, 12, 24}

There are 8 elements in the set F. So, F is finite set.

## 1.1.3 Equivalent and Equal Sets

### Activity 4

The sets are A = {a, b, c} and B= {1, 2, 3}.

What are the number of elements in set A and set B? Are the number of elements in set A and set B equal? Discuss.

Here, the number of elements in set A and set B are equal. So, set A and set B are equivalent sets.

> Two sets are said to be equivalent if the number of elements of the sets are equal. Equivalent sets are written as A~ B.

### Activity 5

Observe the set given below and discuss the questions asked:

A = { △, ○, ▭, □ }

B = { □, △, ○, ▭ }

(a) How many elements are there in set A and set B?

(b) Are the number of element in set A and set B equal?

(c) Are the elements in set A and set B same?

There are four elements in both set A and set B.

Every element of set A are in set B and every element of set B are in set A. So, set A and set B are equal sets.

> (a) Two sets are said to be equal, if the number of elements in the sets are equal and same. In symbol it is written as A= B.
>
> (b) All equal sets are equivalent but all equivalent sets may not be equal sets.

## Example 2

**If A and B are two sets, then answer the following questions:**

A = {natural numbers greater than 1 and less than 4}

B = {prime factors of 6}

(a) Write the set A and set B by listing method.

(b) Write the numbers of elements of the both set A and set B.

(c) Is each element of set A in set B?

(d) Is each element of set B in set A?

(e) What type of sets can be called to set A and set B?

### Solution

Here,

(a) A = {2, 3}, B = {2, 3}

(b) The number of elements of the both set A and set B are 2.

(c) Each element of set A is in set B.

(d) Each element of set B is set A.

(e) Set A and set B are equal sets.

## Exercise 1.1

**1. Write the following sets by listing method and also write the numbers of elements of these sets:**

(a) A = { prime numbers up to 13 }

(b) B = {x: x is a multiple of 4 less than 40}

(c) C = {counting numbers greater than 2 and less than 7}

(d) A = {factors of 20}

2. **Tik (√) if the following sets are empty and cross (×) if they are others than empty.**

(a) The set of natural numbers less than 1

(b) The set of even prime numbers

(c) The set of grade 6 students taller than 6 feet

(d) The set of odd numbers divisible by 2

3. **Separate the finite and infinite set from the following sets. If they are finite write the numbers in elements of these sets:**

(a) A = {odd numbers greater than 20}

(b) P = {prime numbers less than 20}

(c) C = {composite numbers less than 20}

(d) W = {whole numbers less than 1}

4. **State which of the following sets are equal? If they are equal, write them using "=" sign.**

(a) P = {prime numbers from 2 to 9}, Q = {odd numbers from 1 to 8}

(b) A = {natural numbers less than 20 which is divisible by 2}, B = {even number less than 10}

(c) C = {*l*, u, r, e}, D = {r, u, *l*,e},

(d) A = {factors of 16}, B= {natural numbers up to 16 which is divisible by 4}

5. **Separate the equivalent set from the following sets:**

(a) A = {g, o, d}, B = {d, o, t}

(b) A = {a, b, c, …, x}, Y = {1, 2, 3, …, 12}

(c) C = {*l*, i, n, e}, D = {f, i, *l*, e}

d) G = {1, 2, 3, 4, 5}, H = {x: x is a natural number less than 6}

**6. If A = {0, 1, 2, 3}, B = {1, 2, 3}, C = { 0}, D = { }, E = {1, 2, 3} and F = {1} are given sets then answer the following questions:**

(a) Write the number of elements of each set.

(b) Which sets, have equal numbers of elements?

(c) Identify the sets which are not equal even though the number of elements is equal?

(d) Which sets are equal? Write.

7. Are the sets A = {4, 5} and B = {c, d} equivalent? Write with reason.

**8. If A = {a, b, c, d} and B = {a, b, c, d, e} then,**

(a) Are the set A and set B equal?

(b) What should be done to set A and set B for making them equal?

## Project Work

Prepare a list of 10 items that are in your classroom. Create different sets based on the same properties from the same item list. Observe the set, write their name and types and present it in the classroom.

## Answer

Show the answers to your teacher.

## 1.2 Universal Set

**Activity 1**

Divide all the students into appropriate number groups, form the set of the following by discussing in the group.

(a) E = set of even numbers

(b) O = set of odd numbers

(c) S = set of square numbers

(d) C = set of cube numbers

(e) P = set of prime numbers

(f) A = set of composite numbers

Which single set can cover all the qualities or characteristics of the above sets (a to f)?

Can all the above sets be discussed in the set of natural number i.e. N = {1, 2, 3, ... }

Similarly, make the set of girl-student in your class (G), set of boy-student (B) and set of students wearing cap (C).

Which definite set can possess all the qualities or characteristics of these three sets?

Does the set S = {set of total students of the class} contain all the above three sets?

> A certain set is said to be a universal set if all the sets under discussion are the subsets of the certain set. The universal set is denoted by U.

## Activity 2

Be divided into appropriate number of groups, read the dialogue below. Discuss what can be its universal set. Present it in the class.

| | | |
|---|---|---|
| Rupesh | : | Odd numbers up to 13. |
| Norbu | : | It doesn't have the number 16 and counting number greater than 16. |
| Manju | : | Even numbers up to 14. |
| Saddam | : | Does it have the prime numbers too? |

Based on the above discussion, find the universal set that Bimal wrote on the board.

Here, all of the above sets can be discussed on the set of natural numbers up to 15, i.e. (N) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}.

So, the set N can be the universal set here.

## Example 1

**Here U = {natural numbers less than 15} is a universal set. Now, write the following sets by listing method:**

(a) A = {even numbers less than 15}

(b) B = {prime numbers less than 15}

### Solution

Here, U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

(a) A = {even numbers less than 15}

$\therefore$ A = {2, 4, 6, 8, 10, 12, 14}

(b) B = {prime numbers less than 15}

$\therefore$ B = {2, 3, 5, 7, 11, 13}

## Exercise 1.2

1. **If U = {natural numbers less than 25} is a universal set then write the following sets by listing method.**

   (a) A = {square numbers less than 25}

   (b) O = {odd numbers}

   (c) C = {composite numbers}

2. **U = {the set of whole numbers from 5 to 20} is a universal set. Make the following sets by listing method based on the given universal set:**

   (a) E = {even numbers}

   (b) $M_3$ = {multiples of 3}

   (c) P = {prime numbers}

3. **Write an appropriate universal set for the following sets:**

   **A = {2, 4, 6, 8, 10, 12, 14} and B = {1, 3, 5, 7, 9}**

4. **Based on the following discussions among the students of class 7 in a school, find the universal set:**

| | | |
|---|---|---|
| Dolma | : | Considering $M_4$, here $M_4$ = {4, 8, 12} |
| Arya | : | There is no zero in this set. |
| Krishna | : | There is not 13 in this set too. |
| Zakir | : | Ah! It doesn't have fraction either . |
| Solmu | : | There is not any even number in it. |
| Jitu | : | It doesn't have decimal numbers too. |

### Answer

Show the answers to your teacher.

## 1.3 Subset

### Activity 1

Be seated in groups of appropriate number. Consider the following set of tools in an instrument box as a univesal set.

U = {protractor, set square, compass, ruler}

Now, each group make the following sets from the above universal set:

**1. Sets having only one element from universal set**

A = {protractor}, B = {set square}

C = {compass}, D = {ruler}

**2. Sets having two elements**

E = {protractor, set square}

F = {protractor, compass}

G = {protractor, ruler}

H = {set square, ruler}

I = {set square, compass}

J = {compass, ruler}

**3. Sets having three elements.**

K = {protractor, set square, compass}

L = {protractor, compass, ruler}

M = {set square, compass, ruler}

N = {protractor, set square, ruler}

**4. Set having four elements**

O = {protractor, set square, compass, ruler}

**5. An empty set**

P = { }

Now discuss the following questions with your friends based on the above set:

(a) Are the element of set A also the elements of set U?

(b) Are all the elements of sets B, C, D also the elements of set U?

(c) Are all the elements of sets E, F, G, H, I and J also the elements of set U?

(d) Are all the elements of sets K, L, M and N also the elements of set U?

(e) Are all the elements of set O also the elements of set U?

> If all the elements of the first set are also the elements of the second set, then the first set is said to be subset of the second set.

## 1.3.1 Proper and Improper Subsets

### Activity 2

Every student make the one/one set each. e.g. P = {1, 2, 3}

Now, make the subset from the set you formed and discuss the following questions:

A = {1}, B = {2} C = {3}

D = {1, 2}, E = {1, 3} F = {2, 3}

G = {1, 2, 3}, E = { }

(a) What is the subset formed with all the elements of P? What types of subset is this?

(b) What are the subsets made up of some elements of P? What are these subsets called? What types of subset are these subsets?

(c) Can an empty set be called proper subset of any set?

(d) How many subsets can be formed from set P? How many of them are proper subsets and improper subset?

(e) Are equal sets improper subset to each other?

(f) How are the proper subset and improper subsetdenoted in mathematical symbols?

(g) How many subsets can be formed from P?

1. If a subset B is formed by taking some elements from the given set A then the subset B is said to be a proper subset of the given set A. It is written as $B \subset A$.
2. If a subset B is formed by taking all the elements from the given set A then the subset B is said to be an improper subset of the given set A. It is written as $B \subseteq A$.
3. Empty set is proper subset of every non-empty set.
4. Equal sets are improper subset to each other.

### Example 1

**Seprate the proper subsets and improper subsets from the following subsets formed from the universal set given below.**

U = {a, e, i, o, u}

A = {a, e}

B = {e, o, u}

C = {a, e, i, o, u}

**Solution**

Here, subsets A and B are proper subsets of U.

Similarly, subset C is improper subset of U.

### Exercise 1.3

1. **If a set F = {Marigold, Rose, Rhododendron}, then**
   (a) Make the subsets taking only one element from set F and name them.
   (b) Make the subsets taking two elements from set F and name them.
   (c) Make the subset having all elements from set F and name it.
   (d) Make the subset having no element and name it.
2. Separate the proper subset and the improper subset from the subsets formed in Question (1).

3. Among the sets A= {1, 2, 3, 4}, B = {2, 3, 4} and C = {4, 3, 2, 1}, which is the proper subset of A?

4. Make all the possible subsets from the set A = {a. b, c}.

**5. Write down all possible subsets that can be formed from each of the following sets:**

(a) P = {a, b}  (b) Q = {4, 5}  (c) R = {p, q, r}

**6. Copy the following table and fill in the blanks:**

| Sets | Subsets | Proper subsets | Improper subsets |
|---|---|---|---|
| {1} | | | |
| {1, 2} | | | |
| {1, 2, 3} | | | |

7. Write any one set by listing method. Write all the possible subsets of that set. Seperate the proper and improper subsets and present them in the class.

## Project Work

Make a list of items in your classroom. Make the sets by their characteristics. Make all the possible subsets of each of those sets and present in the class.

## Answer

Show the answers to your teacher.

## Miscellaneous Exercise

1. **If set A = {factors of 6} and B = {prime numbers less than 10}, then**

   a) Write the set A and set B by listing method and set-builder method.

   b) Are the set A and set B equivalent or equal sets, why?

2. **If the set L = {natural numbers from 1 to 10 exactly divisible by 2}, M = {multiples of 2 upto 10} and N = {natural numbers from 12 to 16}, then**

   a) Write the set L, M and N by listing method.

   b) Which sets are equal and equivalent? Write with the reason.

3. **If the set A = {even numbers greater than 10} and B = {even numbers less than 10} then,**

   a) Write the set A and set B by listing method and set-builder method.

   b) What are the set A and set B finite or infinite? Why? Write with reason.

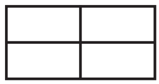
### Answer

Show the answers to your teacher.

# Lesson 2 Whole Number

## 2.0 Review

Look at the pictures, fill out the table given below and discuss.

(i)  (ii)  (iii)  (iv) 

| Figure no. | Number of boxes along the length | Number of boxes along the breadth | Total number of boxes |
|---|---|---|---|
| (i) | | | |
| (ii) | | | |
| (iii) | | | |
| (iv) | | | |

(a) How do we know the total numbers of boxes without counting?

(b) Which pictures have equal number of square boxes along length and breadth?

(c) Which pictures are of square shapes?

(d) What does the total number of boxes in a square figure and the number of boxes along the length denote?

- If a number is multiplied by itself, the product so obtained is called square of that number.
- Any square number has two equal factors, one of those factor is called the square root of that number. It is written using the symbol '$\sqrt{\ }$' to denote the square root of a number.
  For example: $\sqrt{64} = \sqrt{8^2} = 8$.

## 2.1 Square and Square root of the Number

### Activity 1

Take a square number, such as: 64

Now subtract the successive odd numbers starting from 1,3,5,7,9,11 from that number. The number of steps obtained to get the result 0 is the square root of the given number.

Step 1: $64 - 1 = 63$

Step 2: $63 - 3 = 60$

Step 3: $60 - 5 = 55$

Step 4: $55 - 7 = 48$

Step 5: $48 - 9 = 39$

Step 6: $39 - 11 = 28$

Step 7: $28 - 13 = 15$

Step 8: $15 - 15 = 0$

In how many times did the result come zero?

Since the result is zero in the eighth time, the square root of 64 is 8.

### 2.1.1 Square Root by Prime Factorization Method

### Activity 2

Look at the following examples and discuss the questions asked below.

Finding the prime factor of 6 and 36,

| | |
|---|---|
| 2 | 6 |
| | 3 |

| | |
|---|---|
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| | 3 |

$6 = 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

(a) What is the relationship between the number 6 and 36?

(b) How many 2 are there among the prime factors of 6?

(c) How many 3 are there among the prime factors of 6?

(d) How many 2 and 3 are among the prime factors of 36?

(e) What difference did you find between the square number and the prime factors of its square root?

Now, finding the square root of 36,

$= \sqrt{36}$

$= \sqrt{2 \times 2 \times 3 \times 3}$

$= \sqrt{2^2 \times 3^2}$

$= 2 \times 3$

$= 6$

Therefore, the square root of 36 is 6.

**Procedure for finding the square root by prime factorization method:**

- Find the prime factors of the given number,
- Put the prime factors in root ($\sqrt{\ }$) sign,
- Write the number of pairs in exponents form,
- Write one number of each pair outside the root ($\sqrt{\ }$) sign and multiply.
- The product so obtained is the square root of that number.

## Example 1

**Find the square root of 1225 by prime factorization method:**

**Solution**

Here, finding the prime factors of 1225,

$$\begin{array}{r|r} 5 & 1225 \\ \hline 5 & 245 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

Now, finding the square root of 1225,

$= \sqrt{1225}$

$= \sqrt{5 \times 5 \times 7 \times 7}$

$= \sqrt{5^2 \times 7^2}$

$= 5 \times 7$

$= 35$

## Example 2

**If the area of a squared seminar hall is 625 m², find its length**

**Solution**

Here, the area of the squared seminar hall (A) = 625 $m^2$

Length ($l$) = ?

Now,

A = 625 $m^2$

or, $l^2 = 625\ m^2$

or , $l = \sqrt{625}$

$= \sqrt{5 \times 5 \times 5 \times 5}$

$= \sqrt{5^2 \times 5^2}$

$= 5 \times 5$

$= 25$

| | |
|---|---|
| 5 | 625 |
| 5 | 125 |
| 5 | 25 |
| | 5 |

∴ The length of the seminar hall is 25 m.

## 2.1.2 Square Root by Division Method

### Activity 3

**Observe the method of finding the square root of 1764 by division method and discuss:**

(a) How is the digits of 1764 paired?

(b) Why was the number divided by 4 at first?

(c) Why was it added by putting 4 below 4 again?

(d) Should we take the same number in divisor and quotient when dividing always? Find its conclusion.

| | |
|---|---|
| | 4 2 |
| 4 | $\overline{17}\ \overline{64}$ |
| + 4 | – 16 ↓ |
| 82 | 1 64 |
| + 2 | −1 64 |
| 84 | 0 |

**Procedure:**

1. Begin on the right mark off the digits in pairs from ones places or right to left. For example: $\overline{17}\ \overline{64}$
2. Considering the first pair 17 as a square number, the smaller than 17 but the largest perfect square number is 16. Its square root is calculated to be 4.
3. When multiplying by 4 with putting top as quotient and also in the division, the product should be subtracted by putting below 17. Double 4, the number in the quotient and place the double that is 8 as next divisor.
4. Bring down the next pair 64 with the remainder 1. Now the dividend is 164.
5. Now you have to put a number 2 in the quotient to the right of 4, and to the right of 8 in the divisor, multiply the number formed by the added number 82 by 2 and place the product 164 below the dividend (164).
6. Now subtract it, the remainder is 0, If divisible, you can calculate by putting it to the right of the number is large. So, the square root of 1764 becomes 42.

## Example 4

**Find the square root of 955481 by division method.**

**Solution**

Here,

| | |
|---|---|
| | 3 0 9 |
| 3 | 9 $\overline{54}$ $\overline{81}$ |
| + 3 | − 9 |
| 60 | 0 54 |
| + 0 | − 0 |
| 609 | 54 81 |
| + 9 | − 54 81 |
| 618 | 0 |

Therefore, the square root of 95481 is 309.

- Here, pairing from ones places to 9 5481 was $\overline{9}\ \overline{54}\ \overline{81}$ .
- Now the first dividend is 9.
- Dividing 9 by divisor 3, the remainder became 0.
- Bringing down 54 to the right of 0 and it became 054.
- Adding 3 to divisor 3, it became 6.
- Any number placed on the right side of 6 is greater than 54. So put the 0 right side of 6 it becomes 60.
- Multiplying 60 by 0 became 0 which is placed down of 054.
- Subtracting 0 from 054 is 054.
- 81 was brought down to the right of 054.
- Now the dividend was 05481.
- Placing 9 to the right of 60 became 609.
- Multiplying 609 by the added number 9 became 5481. Now subtracting it the remainder become 0, so the calculation was correct. The square root of 95481 became 309.

## Example 5

**Find the square root of $\frac{144}{169}$.**

**Solution**

Here, finding the square root of $\frac{144}{169}$

$$= \sqrt{\frac{144}{169}}$$

$$= \sqrt{\frac{2 \times 2 \times 2 \times 2 \times 3 \times 3}{13 \times 13}}$$

$$= \sqrt{\frac{2^2 \times 2^2 \times 3^2}{13^2}}$$

$$= \frac{2 \times 2 \times 3}{13}$$

$$= \frac{12}{13}$$

- The square root of the numerator and the denominator must be calculated separately.
- The answer should be written with simplifying the fraction.

## Example 6

**Is 12675 a square number? If not so, what is the number by which 12675 is divided to make the quotient square number?**

**Solution**

| | |
|---|---|
| 3 | 12675 |
| 5 | 4225 |
| 5 | 845 |
| 13 | 169 |
| | 13 |

Here, $12675 = 3 \times 5 \times 5 \times 13 \times 13$

Making pairs of the two same numbers,

$2675 = 3 \times 5^2 \times 13^2$

When making pairs of the two same number, there is not pair of 3.

So, 12675 is not a perfect square number. The number that comes from dividing 12675 by 3 is the squared number.

Hence, the required number = 3

## Exercise 2.1

1. **Find the square of the given numbers:**

   (a) 19 (b) 20 (c) 18 (d) 35 (e) 54 (f) 63

2. **Tick (√ ) if the following statements are true and cross (×) if they are false.**

   (a) If any number has 0, 1, 4, 5, 6 and 9 in the ones place, then that number is a square number.

   (b) If there are zeros at the end of any even number, then that number is square number.

   (c) 169000 is perfect square number.

3. **Find the square root of the following numbers by prime factorization method:**

   (a) 169 (b) 324 (c) 1225 (d) 5625 (e) $121 \times 196$

4. **Find the square root of the following numbers by division method:**

   (a) 2304 (b) 8836 (c) 9801 (d) 11025 (e) 95481

5. **Prove that:**

(a) $\sqrt{25} \times \sqrt{36} = \sqrt{25 \times 36}$ (b) $\frac{\sqrt{625}}{\sqrt{25}} = \sqrt{\frac{625}{25}}$

6. **Find the square root of the following numbers:**

(a) $\frac{625}{1024}$ (b) $\frac{49}{81}$ (c) $\frac{324}{1225}$ (d) $\frac{1225}{2916}$

(e) $121 \times 196$ (f) $144 \times 169$ (g) $1\frac{91}{2025}$

7. Is 500 a square number? Test by prime factorization method.

8. Is 325 a square number? If not, find out by which least number should 325 be multiplied to get a perfect square number.

9. Find the area of a squared land having length 37 m.

10. If the area of a squared seminar hall is 729 $m^2$, find its length.

11. The social teacher has taken the class 7 students to visit the Hanumandhoka. The amount for lunch eaten by the student is Rs. 15625. Find out the number of students if all the students have eaten lunch equal to the number of students.

12. What is the square root of 196? Find out from the Procedure of continuous subtraction.

13. Class 7 students have donated the total of Rs 2,500 for flood relief. Each student gave as much as the number of students in the class. Find out how many students are there in the class.

14. What is the least number to be added to 1021 so that the sum will be square?

15. What is the least number to be subtracted from 18227 so that the result will be square?

16. Is 7200 a square number? If not, find out by which least number should 7200 be divided to get the quotient square number.

## Project Work

Make a note of the number of students in your school. Test whether the number is square or not. If it's not a square number, what is the least number by which the number multiplied to make a square number? Find out and present in the class.

## Answer

1 and 2 : Show the answers to your teacher.

3. (a) 13 (b) 18 (c) 35 (d) 75 (e) 154

4. (a) 48 (b) 94 (c) 99 (d) 105 (e) 309

5. Show the answers to your teacher.

6. (a) $\frac{25}{32}$ (b) $\frac{7}{9}$ (c) $\frac{18}{35}$ (d) $\frac{35}{54}$

(e) 154 (f) 156 (g) $1\frac{1}{45}$

7. No

8. No, Multiplying by 13 becomes the square number

9. 1369 $m^2$

10. 27

11. 125

12. 14

13. 50 persons

14. 3

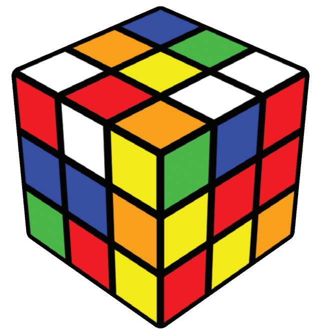
15. 2

16. 2

## 2.2 Cube and Cube Roots of Numbers

**Activity 1**

**Observe the Rubik's Cube in the given picture. How many unit cubes are there in the picture? Discuss.**



On the upper surface: $3 \times 3 = 9$ unit cubes

Being 3 surfaces= $9 \times 3 = 27$ unit cubes

The product of three of the same number is called the cube number, for example, if any number is 3, then it can be written as the cube number of 3 is $3 \times 3 \times 3 = 27$.

**Example 1**

**Find the cube number of 5.**

**Solution**

Here, the cube number of 5 $= 5 \times 5 \times 5$

$= 125$

Here, the cube number of 5 is $= 125$

**Example 2**

**A tank has 6 feet length, 6 feet breadth and 6 feet height which is being built underground to store drinking water in your home. How many cubic feet of underground pit is required to construct the tank?**

**Solution**

Here the cube number of 6 is the required number.

The volume of the underground pit (V) = 6 ft × 6 ft × 6 ft

$= 36\ \text{ft}^2 \times 6$ ft

Therefore, the required volume of the pit is 216 cubic feet.

## 2.2.1 Cube Root by Prime Facorization Method

### Activity 2

Fill in the chart given below and discuss the conclusion:

| Number | Product of multiplying a number by itself three times | Cube number of this number | One of the three same factors |
|---|---|---|---|
| 1 | $1 \times 1 \times 1 = 1$ | 1 | 1 |
| 2 | $2 \times 2 \times 2 = 8$ | 8 | 2 |
| 3 | $3 \times 3 \times 3 = 27$ | 27 | 3 |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |

One of the three same factors of a cube number is the cube root of the number. For example: 2 is cube root of $8 = 2 \times 2 \times 2$ or $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$

### Example 3

**Find the cube root of 512.**

**Solution**

Here, finding the prime factors of 512.

$$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

| | |
|---|---|
| 2 | 512 |
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| | 2 |

Now, cube root of 512,

$= \sqrt[3]{512}$

$= \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{2 \times 2 \times 2}$

$= \sqrt[3]{2^3} \times \sqrt[3]{2^3} \times \sqrt[3]{2^3}$

$= \sqrt[3]{2^3 \times 2^3 \times 2^3}$

$= 2 \times 2 \times 2$

$= 8$

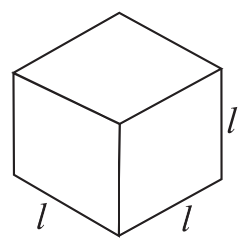
Procedure:

- Find the prime factors of the given number,
- Put the prime factors in the sign $\sqrt[3]{\ }$,
- Collect three of the same number in a group,
- Take one/ one number for each collection and multiply,
- The product obtained is the cube root of that number.

## Example 4

**If the volume of a cubical tank is 4096 m$^3$, find the height of this tank:**

**Solution**




Here, Volume of tank (V) = 4096 m$^3$,

Height of the tank ($l$) = ?

To find the height of the tank, we have to find the cube root of 40961.

| | |
|---|---|
| 2 | 4096 |
| 2 | 2048 |
| 2 | 1024 |
| 2 | 512 |
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| | 2 |

Height of the tank ($l$) $= \sqrt[3]{4096}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= \sqrt[3]{2^3 \times 2^3 \times 2^3 \times 2^3}$

$= \sqrt[3]{2^3} \times \sqrt[3]{2^3} \times \sqrt[3]{2^3} \times \sqrt[3]{2^3}$

$= 2 \times 2 \times 2 \times 2$

$= 16$

Therefore, the height of the cubical tank is 16 m.

## Example 5

**Is 1323 a cubic number? If not, find out by which least number should be multiplied to make it a perfect cube:**

**Solution**

Here, 1323 $= 3 \times 3 \times 3 \times 7 \times 7$

$= 3^3 \times 7^2$

| | |
|---|---|
| 3 | 1323 |
| 3 | 441 |
| 3 | 147 |
| 7 | 49 |
| | 7 |

There are only two 7 among the prime factors of 1323. Another number 7 is needed to get 1323 perfect cube.

Therefore, the required number is 7 which is multiplied by 1323, the product becomes perfect cube number.

## Exercise 2.2

**1. Find the cube of the of the following numbers:**

(a) 7 (b) 12 (c) 13 (d) 15

(e) 21 (f) 30 (g) 42

**2. Find the cube root of the following numbers.**

(a) 8 (b) 343 (c) 1000 (d) 8000

3. What is the volume of a cubic room whose length is 25 m?

4. The Drinking Water Project has constructed a 27-meter-long cubical tank. What is the capacity of the tank? ($1m^3 = 1000\ l$)

5. Is 392 a cube number? If not, find out by which least number should 325 be multiplied to get a cube number. Find by using prime factorization method.

6. Is 1728 a cube number or not? Test by using prime factorization method.

**7. Prove that:**

(a) $\sqrt[3]{27} \times \sqrt[3]{125} = \sqrt[3]{27 \times 125}$ (b) $\sqrt[3]{\frac{512}{64}} = \frac{\sqrt[3]{512}}{\sqrt[3]{64}}$

8. Is 2916 a cube number? If not, find out by which number should 2916 be divided to get the quotient a perfect cube number?

9. Which least number should 3993 be divided to get the quotient is cube number, find out the cube number of quotient.

**10. Find the cube of natural numbers from 1 to 10. Test and write as given below:**

(a) Is the cube number of natural odd numbers also odd?

(b) Is the cube of natural even number also even?

## Project Work

Search and write about at least five examples of cube and cube numbers used in our daily life and present in the class.

## Answer

1. (a) 343 (b) 1728 (c) 2197 (d) 3375
   (e) 9261 (f) 27000 (g) 74088
2. (a) 2 (b) 7 (c) 10 (d) 20
3. 15625 $m^3$ 4. 19683000 *l*
5. No, multiplying by 7 gives a cube number.
6. yes
7. Show the answers to your teacher. 8. 4 9. 3, 11
10. No, multiplying by 7 gives cube number.

## 2.3 Highest Common Factor

### 2.3.0 Review

**Activit 3**

Fill in the blanks given below and discuss:

Factors of 20 = {…………..}

Factors of 35 = {…………..}

Common factors of 20 and 35 = {…………..}

Common factors of 20 and 35 = {…………..}

Greatest common factor of 20 and 35 = …………..

HCF of 20 and 35 = ……………….

The greatest common factor among all the common factors of the given number is called the highest common factor.

### 2.3.1 Methods for Finding Highest Common Factor (HCF)

#### 2.3.1.1 HCF by Prime Factorization Method

Study the following conditions and discuss the questions asked in the group:

**Activity 4**

On her birthday, Harimaya thought to distribute 30 apples and 40 bananas to old fathers and mothers living in old age home.

(a) Find the greatest number of people to whom she can distribute those apples and bananas equally.

(b) How much will each one get?

(c) What will be the mathematical Procedure to find it out?

To do this, you need to find the greatest number that exactly divides the given numbers 30 and 40.

$$\begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline & 5 \end{array}$$

$30 = 2 \times 3 \times 5$

$40 = 2 \times 2 \times 2 \times 5$

$30 \div 10 = 3$

$40 \div 10 = 4$

Product of common prime factors $= 2 \times 5 = 10$

The greatest number that exactly divides 30 and 40 is 10. So Harimaya can distribute apples and bananas equally to 10 people. She can distribute 3/3 apples and 4/4 bananas to each.

The greatest number which divides exactly the given number is HCF

## Example 1

**Find the HCF of the numbers 100, 125, 200 by prime factorization method.**

**Solution**

Here

$$\begin{array}{r|l} 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$100 = 2 \times 2 \times 5 \times 5$

$125 = 5 \times 5 \times 5$

$200 = 2 \times 2 \times 2 \times 5 \times 5$

Product of common prime factors

$= 5 \times 5 = 25$

$\therefore$ HCF = 25

**Procedure**

- Find the prime factors of the given numbers.
- Take common prime factors among them.
- Find the product of common prime factors.
- This product is the required HCF

### 2.3.1.2 HCF by Division Method

**Activity 2**

In the case of given large numbers, it is considered appropriate to find HCF by Division method. In this method, divide the largest number by the smallest number among the given numbers until the remainder is zero.

Observe the following example and discuss how to find HCF by division method in group.

**Which is the greatest number that divides the numbers 100, 125 and 200?**

Here

$$\begin{array}{l} 100)\ 125\ (1 \\ \underline{\quad -100} \\ \qquad 25)\ 100\ (4 \\ \qquad \underline{\quad -100} \\ \qquad\qquad\quad 0 \end{array}$$

Now, dividing the third number 200 by 25.

$$\begin{array}{l} 25)\ 200\ (8 \\ \underline{\quad -200} \\ \qquad\quad 0 \end{array}$$

Therefore, the greatest number that divides the numbers 100, 125 and 200 is 25.

**Procedure**

- Divide greatest number by smaller one of the given numbers into the next largest number. (Until the remainder is smaller than the divisor)
- Next, divide to the divisor by the remainder. Go on repeating the Procedure of dividing the proceeding by the remainder last obtained, till the remainder is zero.
- Then the last, divisor is the required HCF which is exactly divisible to the given numbers.
- Now, we should divide the third number by HCF of other given numbers.

## Example 2

**How many students at most can 125 oranges, 150 sweet lemons, and 225 guavas be divided equally and how many fruits of each will each student get?**

### Solution

Here, the required number is the HCF of 125,150, and 225.

Dividing each number by HCF, the quotient is the number of fruits that each student gets equally.

Here, finding the HCF of 125 and 150,

```
125) 150 (1
   – 125
   -----
      25) 125 (5
        – 125
        -----
            0
```

Now, dividing the another number 225 by the divisor 25

```
25) 225 (9
  – 225
  -----
      0
```

Therefore, HCF is 25.

Therefore, 25 students at most can be divided equally.

Therefore, it can be distributed 125 oranges, 150 sweet lemons, and 225 guavas equally to at most 25 students.

Each student gets $125 \div 25 = 5$ oranges, $150 \div 25 = 6$ mausam and $225 \div 25 = 9$ gauvas.

## Example 3

**Find the greatest number from which subtracting 10, the difference exactly divides 558, 700 and 840.**

**Solution**

Finding the number that divides 558, 700 and 840 exactly.

```
588) 700 (1
   – 588
     112) 588 (5
        – 560
          28) 112 (4
             – 112
                 0
```

again,

```
28) 840 (30
  – 840
      0
```

HCF of the numbers 558, 700 and 840 = 28

Let the required number = x

According to questions,

$x - 10 = 28$

or, $x = 28 + 10$

or, $x = 38$

Hence, the required number = 38

## Exercise 2.3

**1. Find the HCF of the following numbers by prime factorization method:**

(a) 21 and 28 (b) 26 and 52 (c) 9, 18, and 36

(d) 12, 18 and 36 (e) 20, 35 and 55

**2. Find the HCF of the following numbers by division method:**

(a) 144 and 312 (a) 500 and 625 (a) 120, 320 and 480

(a) 80, 90 and 120 (a) 144, 384 and 432

3. How many people at most can 72 copies and 99 pencils be divided equally and how many copies and pencils will each person get? Find.

4. One of the organizations has donated 125 kg pulses, 150 kg seeds and 235 kg rice as relief to Covid. How many families can these foods be distributed equally? How much quantity of pulses, seeds (gedagudui) and rice does each family get?

5. One organization has managed for the distribution of 80 blankets, 90 sweaters and 120 warm jackets for the flood victims. At most, how many families can be divided equally? Find out how many clothes of each type each family receives.

6. On the occasion of his birthday, Prabin distributed 60 apples, 72 oranges and 108 bananas to the adult in old age home. How many adult at most were these fruits distributed equally? Find out how many fruits each person received.

7. Which is the greatest number that can divide 275, 440 and 715 exactly?

8. Find the greatest number to which adding 10, the sum divides 225, 375 and 525 exactly.

### Answer

| | | | | | |
|---|---|---|---|---|---|
| 1. | (a) 7 | (b) 26 | (c) 9 | (d) 6 | (e) 5 |
| 2. | (a) 24 | (b) 125 | (c) 40 | (d) 10 | (e) 48 |

3. 9 people, 8 copies, 11 pencils
4. 25 families, 5 kg pulses, 9 kg seeds and 11 kg rice
5. 10 families 10 blankets, 90 sweaters and 120 warm jackets
6. 12 people, 5 apples, 6 oranges, 9 bananas
7. 55  8. 74

## 2.4 Lowest Common Multiple

### 2.4.0 Review

#### Activity 1

**Fill in the blanks given below and discuss:**

(a) The set of multiples of 6 = {…………..}

The set of multiples of 8 = {…………..}

The set of common multiples of 6 and 8 = { …………..}

The set of least common multiples of 6 and 8 = { …………..}

(b) Do the numbers 6 and 8 divide the least common multiple exactly? Test by dividing.

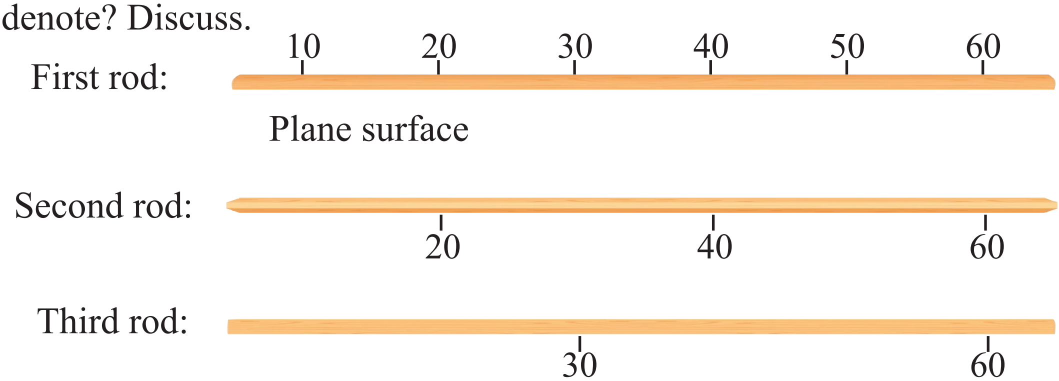
LCM defines the least number which is exactly divisible by two or more numbers. Or

LCM of two or more numbers is the least number that can be exactly divisible by those numbers.

## Activity 2

There are three long rods with the length of 10 cm, 20 cm and 30 cm in the given figure.


As the three rods are measured together on a plane surface, how many cm. of shortest distance can be measured? What does this distance denote? Discuss.




$M_{10} = \{10, 20, 30, 40, 50, 60, ...\}$

$M_{20} = \{20, 40, 60, ...\}$

$M_{30} = \{30, 60, ...\}$

The shortest distance measured by all three rods is 60 cm. It is called LCM

Therefore, LCM = 60 cm

## 2.4.1 Methods for Finding Lowest Common Multiple (LCM)

### Method 1: LCM by Prime Factorization Method

## Activity 1

Discuss about the questions asked by observing the given examples:

## Example 1

**Find the LCM of 18, 24 and 36 by prime factorization method.**

(a) Find the prime factors of the numbers 18, 24 and 36.

(b) Find the common factor of all the numbers. Then find the common factor of 2/2 numbers.

(c) Also, take the remaining prime factors.

(d) Find the product of the common and the remaining prime factors.

(e) What is called to this product? Write the conclusion.

**Solution**

$$\begin{array}{r|r} 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|r} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|r} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

| | | | | | |
|---|---|---|---|---|---|
| Factors of 18 = | 2 × | | | 3 | × 3 |
| Factors of 24 = | 2 × | 2 × | 2 × | 3 | |
| Factors of 36 = | 2 × | 2 × | | 3 | × 3 |

Common factors of 18, 24 and 36 $= 2 \times 3 = 6$

Common factors of 18 and 36 $= 3$

Common factors of 24 and 36 $= 2$

Remaining factors $= 2$

LCM= Common factors × Remaining factors

$= 6 \times 3 \times 2 \times 2$

$= 72$

Therefore, LCM of the numbers 18, 24 and 36 = 72

## Method 2: LCM by Division Method

By combining the given numbers 18, 24 and 36, the LCM can be found in the following methods according to the Procedure of prime factorization:

**Solution**

Here,

| | |
|---|---|
| 2 | 18, 24, 36 |
| 3 | 9, 12, 18 |
| 3 | 3, 4, 6 |
| 2 | 1, 4, 2 |
| | 1, 2, 1 |

$\therefore$ LCM = $2 \times 3 \times 3 \times 2 \times 2$

= 72

**Procedure**

- Write all the given numbers in rows using commas in any order.
- Continue dividing the smallest common prime factor which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible.
- Keep dividing if it divides exactly only two numbers too.
- This Procedure is continued till none of these two of the given number are divisible by a same number.
- Find the product of all divisors prime factors and the remaining factors of the last row. This product is LCM of the given numbers .

### Example 2

**Find the least number which is exactly divisible by 24, 36 and 56**

**Solution**

Here, The least number which is exactly divisible by the given number is LCM of the given numbers.

Finding the LCM of 24, 36 and 56,

| | |
|---|---|
| 2 | 24, 36, 56 |
| 2 | 12, 18, 28 |
| 2 | 6, 9, 14 |
| 3 | 3, 9, 7 |
| | 1, 3, 7 |

$\therefore$ LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 7$

= 504

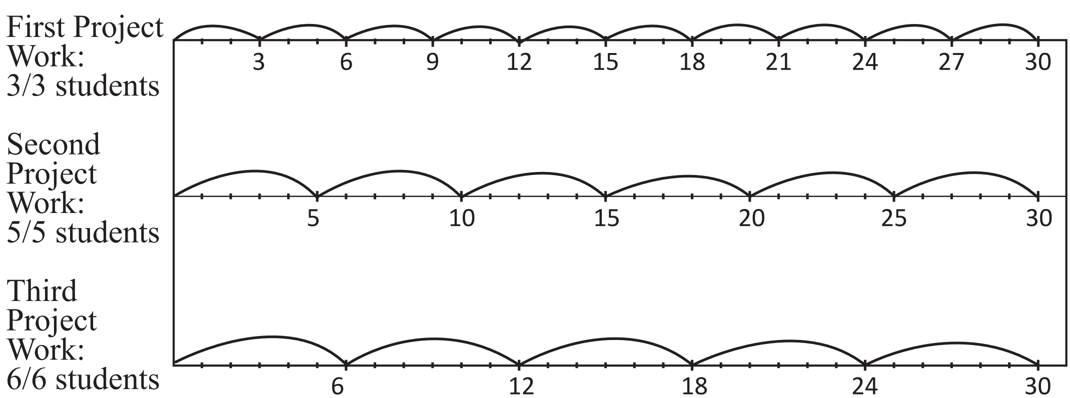
Therefore, 504 is the least number which is exactly divisible by 24, 36 and 56.

## Example 3

**The mathematics teacher has decided to assign 3 Project Works to the students of class 7. When the first Project Work is assigned into the groups of 3 students, the second into the groups of 5 students and the third into groups of 6 students, there are no students left. Now, find out at least how many students are there in the class 7.**

**Solution**

Here:




Minimum number of students required to form a group so that there is no students remained for each task = Project Work can be done by forming a group of 30 students.

$\therefore$ LCM = 30

$\therefore$ There are at least 30 students in class 7.

**Another Method**

Here,

The number of students in each group in the three Project Works is 3, 5 and 6 respectively,

Now, finding the LCM of 3, 5 and 6,

$$\begin{array}{r|l} 3 & 3, 5, 6 \\ \hline & 1, 5, 2 \end{array}$$

$\therefore$ LCM = $3 \times 5 \times 2 = 30$

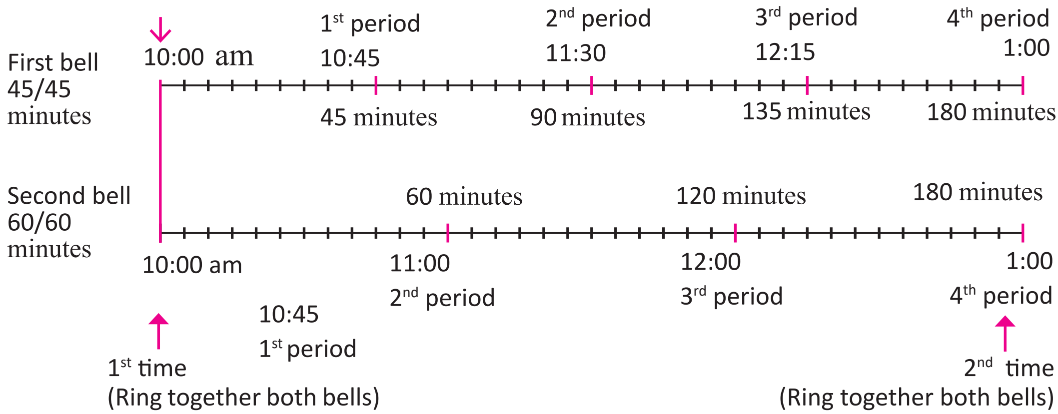
Therefore, if there are at least 30 students, the first Project Work can be done in groups of 3/3 students, the second Project Work in groups of 5/5 students and the third Project Work in groups of 6/6 students.

## Example 4

**Two bells ring at an interval of 45 minutes and 60 minutes. If two bells ring at the same time at 10:00 am first, find out what time they will ring together the next time?**

### Solution

Here,




∴ The second time, at 1:00 o'clock, both bells will ring at the same time.

### Another Method

Here, for this, finding the LCM of 45 and 60,

| | |
|---|---|
| 5 | 45, 60 |
| 3 | 9, 12 |
| | 3, 4 |

$\therefore$ LCM $= 5 \times 3 \times 3 \times 4$

$= 180$ minutes

$= 3$ hours

The second time, the bell rings together = 10:00 o'clock + 3 hours

= 1 o'clock

## Example 5

**Find the least number from which when 5 is subtracted, the difference will be exactly divisible by 18, 24 and 36:**

### Solution

Here

The LCM of 18, 24 and 36 is the number obtained by subtracting 5 from the required number.

| | |
|---|---|
| 2 | 18, 24, 36 |
| 3 | 9, 12, 18 |
| 3 | 3, 4, 6 |
| 2 | 1, 4, 2 |
| | 1, 2, 1 |

So, finding the LCM of 18, 24 and 36.

$\therefore$ LCM $= 2 \times 3 \times 3 \times 2 \times 2$

$= 72$

According to the question, let the required number $= x$, then,

$x - 5 = 72$

or $x = 72 + 5$

or $x = 77$

Hence, the required number is 77.

## Example 6

**Find the least number, with which when 5 is added, the sum will be exactly divisible by 32, 64 and 192.**

### Solution

Here, the LCM of 32, 64 and 192 is the number obtained by adding 5 in the required number.

So, finding the LCM of 32, 64 and 192,

$\therefore$ LCM = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

= 192

| | |
|---|---|
| 2 | 32, 64, 192 |
| 2 | 16, 32, 96 |
| 2 | 8, 16, 48 |
| 2 | 4, 8, 24 |
| 2 | 2, 4, 12 |
| 2 | 1, 2, 6 |
| | 1, 1, 3 |

Let the required number = x

According to question,

$x + 5 = 192$

or $x = 192 - 5$

or $x = 187$

Hence, the required least number is = 187

## Exercise 2.4

**1. Find the LCM of the following numbers by prime factorization method:**

(a) 28, 42 and 56 (b) 3, 11 and 33

(c) 6, 15 and 21 (d) 20, 36 and 44

(e) 50, 75 and 125 (f) 210, 280 and 420

2. Find the least number which is exactly divisible by 42, 49 and 63.

3. The sports teacher divided the students into groups of 15 in the first game, 20 in the second game and 25 in the third game. Find out the least number of students from which the group for each game can be formed without leaving any students.

4. In the three watches, the alarm rang at the interval 10, 15 and 20 minutes respectively. All the alarms rang at 10 o'clock in the morning. Find out at what other time the alarm of the three watches will ring again at the same time.

5. Find the least number from which when 10 is subtracted, the difference will be exactly divisible by 15, 22 , 45.

6. Find the least number, with which when 3 is added the sum is exactly divisible by 8, 12 and 14.

## Project Work

If the paper strip or the rod is of the size 3 units, 11 units and 33 units, what is the LCM of 3, 11 and 33? Show and present it in the class.

## Answer

| | | | | | |
|---|---|---|---|---|---|
| 1. | (a) 168 | (b) 33 | (c) 210 | (d) 1980 | |
| | (e) 750 | (f) 840 | | | |
| 2. | 882 | 3. 300 | 4. 11 o'clock | 5. 190 | 6. 165 |

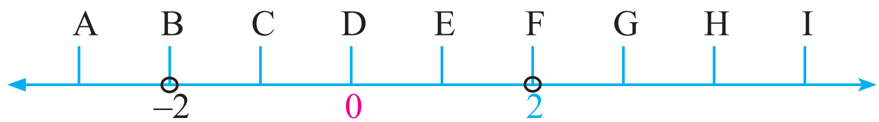
Lesson 3

# Integer

## 3.0 Review

Study the condition given below and discuss it in the group.

(a)




-2, 0 and 2 are represented by B, D and F in the number line. Which integers are represented by A, C, G and I?

(b) How can integers 8, -2, 3, 0 and 6 be represented in the number line?

(c) How can 5 units to the right of place A and 5 units to the left of B be shown from the origin by considering any one point as the point of origin in the number line?

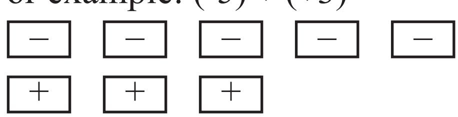
## 3.1 Addition of Integer

### Activity 1

- Take two rectangular pieces of paper of different colors. Write '+' sign on a piece of 1st type colored paper. Write '– ' sign on pieces of 2nd colored paper.

- Take any two numbers from the set of integers and what is the sum? Find out by using the pieces of paper.

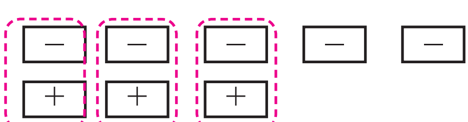
  For example: (-5) + (+3)




Number of '–' is 5.

Number of '+' is 3.

- Now, remove the piece of paper with written '+' and '–' by making pairs.




Two pieces of paper remained consisting '–' sign.

So, it became $(-5) + (+3) = -2$.

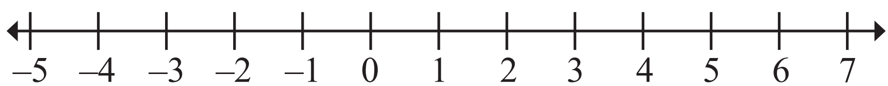
## Activity 2

## Addition of integers through the number lines

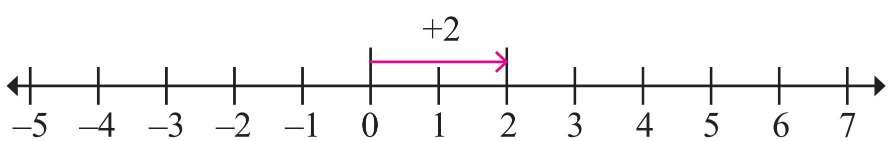
Discuss in groups by observing the number lines given below:

**(a) (+2) + (+4) = ?**

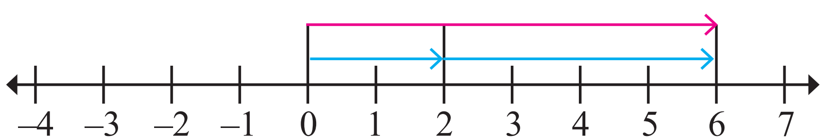
(i) Draw the number line.




(ii) Move 2 units to the right from the origin (zero).




(iii) Now, move 4 units to the right from that point.




(iv) At what number did you reach? Note down in your copy.

(+2) + (+4) = 6

**(b) (+6) + (–2) = ?**

Moving 6 units to the right from the origin and returning 2 units to the left from that point, which point is reached? Note down by making the number line.

**(c) (+2) + (–7) = ?**

Moving 2 units to the right from the origin and 7 units to the left from that point, which point is reached? Note down by making the number line.

**(d) (–3) + (–5) = ?**

Moving 3 units to the left from the origin and 5 units to the left again from that point, which point is reached? Show it in the number line.

## Properties of Addition of Integers

### 1. Closure Property

Take any two numbers from the set of integers Z= {…, –3, –2, –1, 0, 1, 2, 3, …}. Find the sum of these numbers. What is the result? Discuss.

For example, $0 + 1 = 1$
$-2 + 1 = -1$
$-2 - 3 = -5$

The sum of any two integers is also an integer.

If a and b are two integers, then a+b is also an integer. This is called the closure property of addition.

### 2. Commutative Property

Take any two numbers from the set of integers. Change the order of the numbers and find the sum. Discuss what the result is.

For example, $2 + 3 = 3 + 2 = 5$
$-1 + 1 = 1 - 1 = 0$
$-2 + (-3) = (-3) + (-2) = -5$

If we add the integers in any order, then the result becomes the same integer.

If a and b are any two integers, then $a + b = b + a$. This is called Commutative law of addition.

## 3. Associative Property

Take any three numbers from the set of integers. Put the numbers in any order and add up the sum of the first two integers and add the third integer to the sum. What is the result? Discuss.

For example, In –3, –2 and –5

$[(-3) + (-2)] + (-5) = (-3) + [(-2) + (-5)]$

$-5 - 5 = -3 - 7$

$\therefore\ -10 = -10$

If a, b and c are any integers then there is $(a+b) + c = a + (b+c)$ . This is called associative property of addition.

## 4. Additive Inverse

$-5 + (+5) = ?$

$(+2) + (-2) = ?$ Discuss.

If the sum of any two integers is zero, then they are additive inverse to each other.

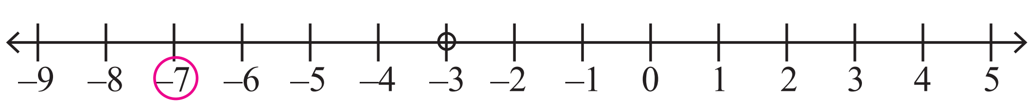
For any integer a, its negative integer is (–a), where $a + (-a) = (-a) + a = 0$. So, a and –a are additive inverse to each other.

### Example 1

**Write the integer which lies 4 unit left from –3.**

**Solution**

Here,




The integers which lies 4 unit left from –3 is (–7).

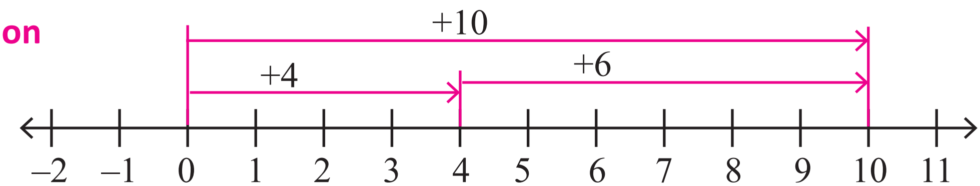
## Example 2

### Add by using number line:

(a) (+4) + (+6)

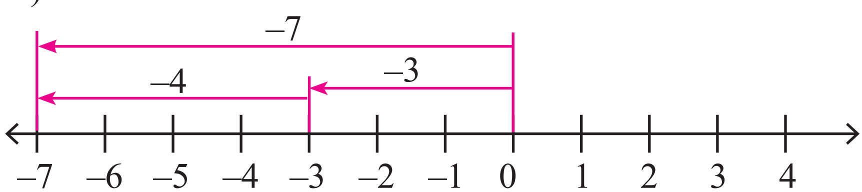
**Solution**

Here,




Therefore, (+4) + (+6) = +10

(b) (–3) + (–4)




Therefore, (–3) + (–4)

= –3 –4

= –7

## Example 3

### Test the associative law from the integers (-7), (-2) and (+6) :

**Solution**

Here,

(–7) + (–2) + (+6)

= [(–7) + (–2)] + (+6)

= –9 + 6

= –3

Again,

(–7) + [(–2) + (+6)]

= (–7) + (+4)

= –7 + 4

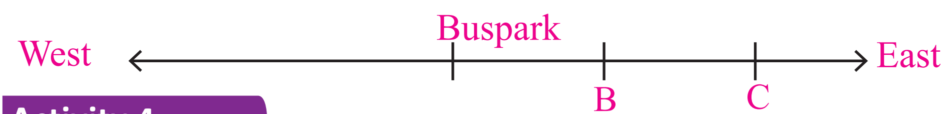
= –3

Therefore, [(–7) + (–2)] + (+6) = (–7) + [(–2) + (+6)] = –3

## 3.2 Subtraction of Integers

### Activity 3

Ram went to the place C to the east 20 km from the bus park. Then he returned 13 km west to the place B from the same route.
If the bus park is to be considered as origin and the distance to the east is taken as positive value, how many kilometers is Ram away from the bus park? Discuss by observing the given number line.

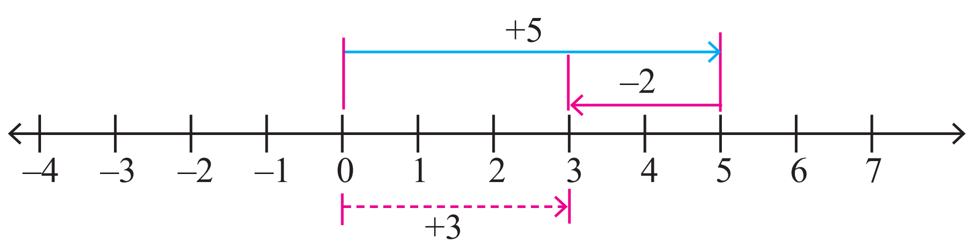



### Activity 4

Observe and discuss about the examples of subtraction of integers from the number line given below:

**1. (+5) – (+2) = ?**

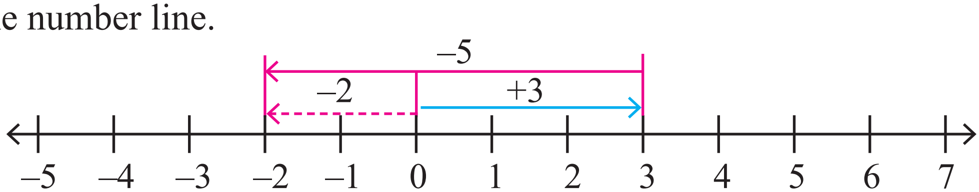
Moving 5 units from 0 (zero) to the right and again 2 units to the left, in which point is reached? Note down from the number line.




We reach at the point 3 unit right or (+3).
Therefore, (+5) – (+2) = (+3)

**2. (+3) – (+5) = ?**

Move 3 units to the right from the origin (0). Return 5 units to the left again from that point. In which point is reached? Discuss by drawing the number line.



We reach at the point 2 unit left or (–2).
Therefore, (+3) – (+5) = (–2)

## Example 4

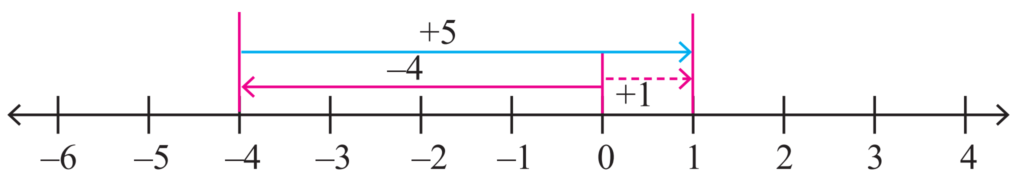
### (a) Show the number (–4) – (–5) in the number line.

**Solution**

Here, (–4) – (–5)

= (–4) +5

= 1




Therefore, (–4) – (–5) = (+1 )

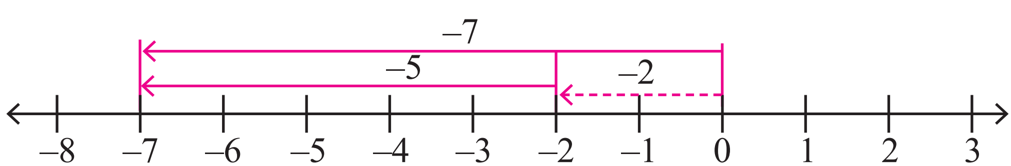
### (b) Show the number (–2) – (+5) in the number line.

**Solution**

Here, (–2) – (+5)

= –2 – 5

= –7




∴ (–2) – (+5) = –7.

**Example 5**

**Simplify:**

(+250) + (–275) – (+148) + (+207) – (175)

**Solution**

Here, (+250) + (–275) – (+148) + (+207) – (175)

= 250 – 275 – 148 + 207 – 175

= (–25) + 59 – 175

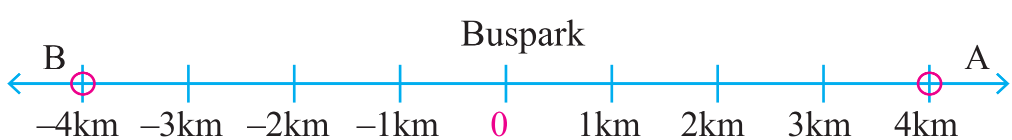
= –25 + 59 – 175

= 34 – 175

= –141

## 3.3 Absolute Value of Integers

Discuss by observing the number line given below:




Here, the bus park is at origin. There is a place A 4 km right from the bus park and there is place B which is 4 kilometer left from the bus park. What is the distance from the place A to the place B?

Is 4 km + (–4) km = 0? Discuss.

The distance from place A to B is 4 km + 4 km = 8 km. Distance is never negative. Therefore, the absolute value of both +4 and –4 is 4. –4 and 4 are opposite of integers.

The positive numerical value of any integer is called absolute value. So, $|+a| = |-a| = a$ .

The integer lying at the same distance from origin but in opposite direction from the origin in which the given integer lie, is called the opposite of the given integer.

## Exercise 3.1

**1. Write the integers which lie 8 unit right from the given number:**

(a) (–2) (b) (–6) (c) 0 (d) (+3) (e) (+5)

**2. Write the integers which lie 8 unit left from the given number:**

(a) (–3) (b) (–4) (c) 0 (d) (+2) (e) (+7)

**3. Write the opposite integer of the given integers:**

(a) (+3) (b) (+4) (c) (–8) (d) 0 (e) (–2)

**4. Write the absolute value of the given integers:**

(a) |+10| (b) |–6| (c) |–5| (d) |+4| (e) |–9|

**5. Find the sum by using number lines:**

(a) (+3) + (+4) (b) (–4) + (–3)

(c) (+5) + (–2)(d) (–5) + (+2)

**6. Subtract by using number lines:**

(a) (–8) – (–3) (b) (+9) – (–4)

(c) (+4) – (+5) (d) (+7) – (+2)

**7. Find the sum from the both ways using the commutative law of the integers given below.**

(a) (+3) and (+6) (b) (+4) and (–3)

(c) (+5) and (–3) (d) (–3) and (–1)

**8. Find the sum from the both ways using the associative law of the integers given below.**

(a) (+2), (–3) and (–5) (b) (+4), (–3) and (+6)

(c) (–5), (+4) and (0) (d) (–2), (–5) and (+8)

9. What is the sum of (+9) and its opposite integer?

10. What is the sum of (–30) and (+30)?

11. What should be subtracted from (–25) to get (–20)?

12. The sum of two integers is (–115), if one integer is 175. Find the next integer.

13. Two buses departed from the same place at the same time. One bus traveled 125 kilometers to the east and the other 120 kilometers to the west. Find the distance traveled by the two buses.

**14. Simplify:**

(a) (–30) – (–40) – (–20) + (+2)

(b) (+75) – (–14) – (–10) + (+1)

(c) (–40) + (–25) + (+60) + (–5)

(d) (–30) – (–40) – (–20) – (–10)

15. Find the sum of each rows, each columns and the diagonals from the two tables given below. Which table gets the results in the same sum of rows, columns and diagonals? Write.

Table 1

| | | |
|---|---|---|
| –5 | –1 | –4 |
| –5 | –2 | 7 |
| 0 | 3 | –3 |

Table 2

| | | |
|---|---|---|
| 1 | –10 | 0 |
| –4 | –3 | –2 |
| –6 | 4 | –7 |

## Answer

1. (a) +6 (b) +2 (c) +8 (d) +11 (e) +13
2. (a) –11 (b) –12 (c) –8 (d) –6 (e) –1
3. (a) –3 (b) –4 (c) +8 (d) No opposite integer (e) +2
4. (a) 10 (b) 6 (c) 5 (d) 4 (e) 9

5-8. Show the answers to your teacher.

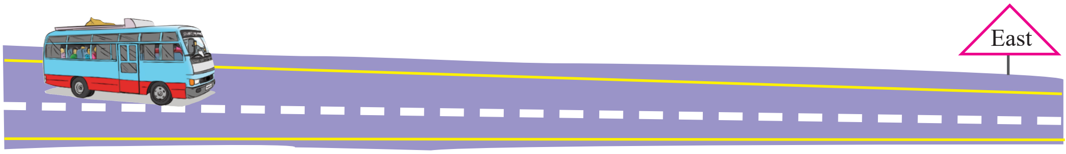
9. 0 10. –60 11. –5

12. –290 13. 245 km 14. (a) +32 (b) +100 (c) –10

(d) +40 15. Show the answers to your teacher.

## 3.4 Multiplication of Integers

Discuss by observing the following condition:




How much distance can a bus travel in 8 hours from west to east at the rate of 40 km per hour?

The distance covered by the bus in 8 hours = 8 × 40 km

= 320 km

### Activity 1

Complete the multiplication table given below. What is the product of the number asked? Find out by discussing.

| × | 3 | 2 | 1 | 0 | –1 | –2 | –3 |
|---|---|---|---|---|---|---|---|
| 3 | 9 | 6 | 3 | 0 | –3 | –6 | –9 |
| 2 | 6 | 4 | 2 | 0 | –2 | –4 | –6 |
| 1 | 3 | 2 | 1 | 0 | –1 | –2 | –3 |
| 0 | | | | | | | |
| –1 | | | | | | | |
| –2 | | | | | | | |
| –3 | | | | | | | |

(a) (–3) × (–2) = ______

(b) (+2) × (–1) = ______

(c) (+1) × 0 = ______

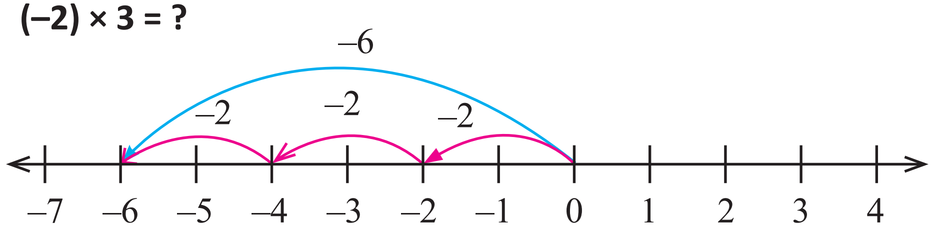
**Having same sign:** →
- Positive integer × positive integer = positive integer
- Negative integer × negative integer = positive integer

**Having opposite sign:** →
- Positive integer × negative integer = negative integer
- Negative integer × positive integer = negative integer

## Activity 2 Multiplication by using number lines

Observe and discuss the multiplication of numbers using the number line given below.

**(a) (–2) × 3 = ?**




Which point is reached by moving 2 units left 3 times from the origin?
Discuss:

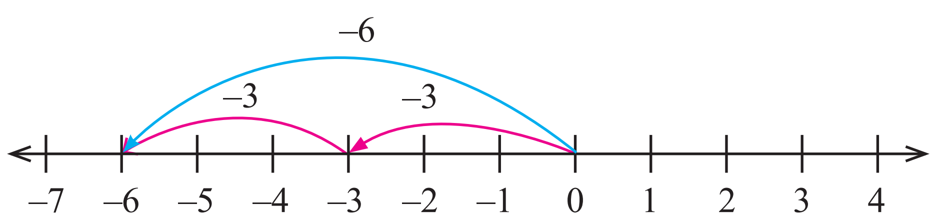
Here, $(-2) \times 3$

$= -6$

Hence, $(-2) \times 3 = -6$

**(b) 2 × (–3) = ?**

Go 3 units left 2 times from the origin. At what point do you reach? Note down by drawing number line.




Here,

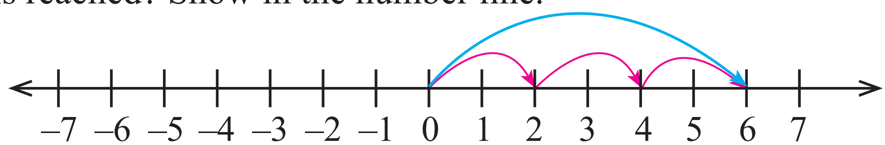
$2 \times (-3)$

$= -6$

Hence, $2 \times (-3) = -6$

**(c) (–2) × (–3) = ?**

(–2) × 3 represents the point reached by moving 2 units with 3 times to the left from the origin as in activity 3 (a). The '–' sign in front of 3 means to go in the opposite direction of the previous direction. So, count how many units there are from the origin to the point given by (–2) × 3 and go the equal unit to the right from the origin. In which point is reached? Show in the number line.




Here, (–2) × (–3)

= 6

Therefore, (–2) × (–3) = 6

## Properties of Multiplication of Integers

### 1. Closure Property

Take any two numbers from the set of integers. Find the product of the numbers. Discuss what the result is.

> The product of any two integers is also an integer. It is called closure property of multiplication.
> If a and b are two integers, then $a \times b$ is also integer.

For example: (–5) × (+4) = (–20)
(–4) × (–2) = (+8)
1 × 0 = 0

### 2. Commutative Property

Take any two numbers from the set of integers. Find the product by changing the order of those numbers. What is the result? Discuss.

For example: 3 × 2 = 2 × 3 = 6
(–8) × (+3) = (+3) × (–8) = –24
1 × 0 = 0 × 1 = 0

> If the product of any two integers is equal to the product of their changed order, it is called commutative property of multiplication.
> If a and b are two integers, then $a \times b = b \times a$.

## 3. Associative Property

Place any three integers in different order. Find the product of the first two and multiply by the third. What comes the product? Discuss.

For example, In three integers 2, 3 and – 4,

$[2 \times 3] \times (-4) = 2 \times [3 \times (-4)]$

$6 \times (-4) = 2 \times (-12)$

$\therefore\ -24 = -24$

The product of three integers remains unchanged even if the order in which they are grouped is altered.

If $a$, $b$ and $c$ are three integers, then
$(a \times b) \times c = a \times (b \times c)$

## 4. Distributive Property

For example, in three integers (+6), (+3) and (–2),

$+6\ [(+3) + (-2)] = (+6) \times (+3) + (+6) \times (-2)$

or, $+6\ (+1) = 18 - 12$

or, $+6 = +6$

If $a$, $b$ and $c$ are three integers, then
$a(b + c) = a \times b + a \times c$

## 5. Multiplicative Property of 1

$(-5) \times 1 = (-5)$

$1 \times (+6) = +6$

If $a$ be a integers, then
$a \times (+1) = (+1) \times a = a$

## 6. Multiplicative Property of Zero

$2 \times 0 = 0$

$0 \times 2 = 0$

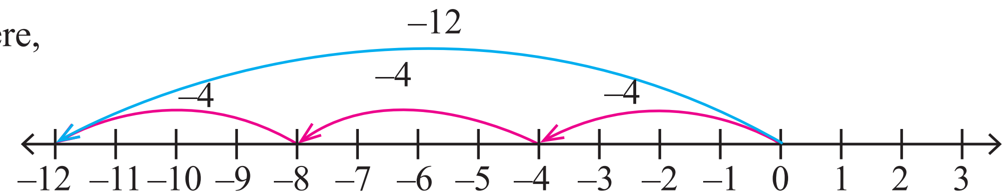
If $a$ be a integers, then $a \times 0 = 0 \times a = 0$

## Example 1

**Multiply by using number line: $(-4) \times 3$.**

**Solution**

Here,




Therefore, $(-4) \times 3 = (-12)$

## Example 2

**Multiply $(+12) \times (-8) \times (+2)$**

**Solution**

Here, $(+12) \times (-8) \times (+2)$

$= (-96) \times (+2)$

$= -192$

## Example 3

**Multiply (+5), (+6) and (–7) from both ways using the Associative property of multiplication.**

**Solution**

Here, $(+5) \times (+6) \times (-7)$

$= [(+5) \times (+6)] \times (-7)$

$= (+30) \times (-7)$

$= (-210)$

Again, $(+5) \times [(+6) \times (-7)]$

$= (+5) \times (-42)$

$= (-210)$

$\therefore\ [(+5) \times (6)] \times (-7) = (+5) \times [(6) \times (-7)] = (-210)$

## Example 4

**Simplify using the distributive property of multiplication:**

(a) $(-5) \times [(+24) - (-6)]$

**Solution**

Here, $(-5) \times [(+24) - (-6)]$

$= (-5) \times (+24) - (-5) \times (-6)$

$= (-120) - (+30)$

$= -120 - 30$

$= -150$

$\therefore\ [(-5) + [(+24) - (-6)] = -150$

## 3.5 Division of Integers

Discuss by observing the following examples:

$(-8) \times (-4) = 32$

$32 \div (-8) = ?$

$32 \div (-4) = ?$

Here is $32 \div (-8) = (-4)$ and $32 \div (-4) = (-8)$

| | | | |
|---|---|---|---|
| **Having same sign:** | Positive integer ÷ positive integer | = | positive integer |
| | Negative integer ÷ negative integer | = | positive integer |
| **Having opposite sign:** | Positive integer ÷ negative integer | = | negative integer |
| | Negative integer ÷ positive integer | = | negative integer |

## Exercise 3.2

**1. Fill in the blanks with appropriate number:**

(a) $\square \div (-6) = 4$

(b) $81 \div \square = (-9)$

(c) $19 \times \square = 0$

(d) $-20 \times \square = -20$

(e) $\square + (-45) = 1$

**2. Multiply by using number line:**

(a) $(+3) \times (+2)$ (b) $(-5) \times (+3)$

(c) $(+2) \times (-6)$ (d) $(-4) \times (-3)$ (e) $(+5) \times (-4)$

**3. Find the product in two different way, using the associative property:**

(a) $(+3) \times (+4) \times (+5)$ (b) $(+7) \times (-5) \times (-3)$

(c) $(-2) \times (-2) \times (-2)$ (d) $(+4) \times (+8) \times (-5)$

**4. Simplify by using the distributive property of multiplication:**

(a) $(+6) \times [(-8) + (+30)]$

(b) $(-9) \times [(+24) - (-6)]$

(c) $(+7) \times [(-12) - (+8)]$

(d) $(-8) \times [(-3) + (-5)]$

**5. Find the quotient:**

(a) $(+36) \div (+6)$ (b) $(-45) \div (+5)$

(c) $(+54) \div (-6)$ (d) $(-95) \div (-1)$

**6. Simplify the given numbers.**

(a) $[(+7) \times (+8) \times (-6)] \div (-3)$

(b) $[(+12) \times (-8)] \div [(+2) \times (-1)]$

(c) $[(+6) \times (+4)] \div [(-3) \times (-2)]$

(d) $(+5) \times (-4) \times (-8) \times (-3)$

7. The product of two integers is (+63). If one of them is (+7), find the other integer.

8. Find the number for which if multiplied by (–5) the product is 90.

9. Find the number which divides (+56) and the quotient is (+70), find.

10. How many times is (–12) multiplied to get the product (–144)?

11. How many times is (–13) multiplied to get the product (–169)?

**12. In a quiz contest, a rule is made to provide (+5) for each correct answer, (–2) for each wrong answer and (0) if not answered.**

(a) If group A gave 4 correct answers and 5 incorrect answers, how much marks did they get?

(b) How much marks did group B get if they gave 5 correct answers and 5 incorrect answers?

(c) Which group scored the highest marks? How much more marks did the group obtain? Find out.

**13. In a quiz contest, rules are made to provide (+3) for each correct answer and (–2) for each incorrect answer.**

(a) Group A scored 18. if they answered 12 questions incorrectly, find out how many questions they answered correctly?

(b) Group B obtained (–5) marks. If they answered 7 questions were answered incorrectly and find out how many questions they answered correctly.

(c) Which group answered most of the questions correctly? How many more questions did the group answer?

## Project Work

Show tthe mathematical operation between the integers given below in number line and present it in the class.

1. $(+3) \times (+2)$
2. $(+3) \times (-2)$
3. $(-3) \times (+2)$
4. $(-3) \times (-2)$

## Answer

| | | | | |
|---|---|---|---|---|
| 1. (a) –24 | (b) –9 | (c) 0 | (d) +1 | (e) +46 |

2-4. Show the answers to your teacher.

| | | | |
|---|---|---|---|
| 5. (a) +6 | (b) –9 | (c) –9 | (d) +95 |
| 6. (a) +112 | (b) +48 | (c) +4 | (d) –480 |
| 7. +9 | 8. –18 | 9. +8 | 10. 12 11. 13 |
| 12. (a) 10 | (b) 15 | (c) group B, 5 | |
| 13. (a) 14 | (b) 3 | (c) group A, 11. | |

## 3.6 Simplification of Integers

We have already discussed in the previous lesson the problems of addition, subtraction, multiplication and division of integers. Now, we will discuss about the simplification of integers.

**Activity 1**

Discuss by observing the questions asked about the mathematical problem given below.

In a quiz contest, rules are made for each correct answer (+5), and for each incorrect answer (–3).

(a) If annapurna house won with a total of 104 marks, in which 25 questions were answered correctly, how many questions did they answer incorrectly?

(b) What mathematical operations should be done to solve this problem?

Here, writing the given problem in mathematical sentences,

$$25 \times (+5) + \boxed{\phantom{00}} \times (-3) = 104$$

Let, number of questions answered incorrectly = $x$

Now, $25 \times (+5) + x \times (-3) = 104$

or, $125 - 3x = 104$

or, $-3x = 104 - 125$

or, $-3x = -21$

$\therefore x = 7$

7 questions were answered incorrectly.

## Example 1

**Simplify:**

$(+12) + (–5) + (+25) \div (–5) – (–6) \times (+7)$

$= (+12) + (–5) + (–5) – (–6) \times (+7)$

$= (+12) + (–5) + (–5) – (–42)$

$= (+7) + (–5) + 42$

$= +2 + 42$

$= 44$

- The problem consisting addition, subtraction and multiplication signs, first treat the operation of multiplication.
- The problem consisting addition subtraction and divisions signs, first treat the operation of division.
- The problem consisting multiplication and division signs, first treat the operation which comes first from left to right.

## Exercise 3.3

**1. Simplify:**

(a) $(–6) \times (–4) \div (+4) + (–5) – (–1)$

(b) $(–15) \div (+5) \times (–4) + (–10) – (+7)$

(c) $(–12) + (+16) \times (–27) \div (–9)$

(d) $(–3) \times (+16) – \{(+12) \div (+6) + (–10)\}$

**2. In a school quiz contest, rules were made to provide (+10) for each correct answer and (–5) for each incorrect answer.**

(a) If the Blue House scored 60 marks, in which 2 questions were answered incorrectly then how many questions were answered correctly? Find out.

(b) If the Yellow House obtained 20 marks, in which 4 questions were answered incorrectly, how many questions were answered correctly? Find out.

(c) Which group answered more questions correctly? How many more questions did the group answer correctly than the group who answered less? State.

3. At the beginning of the month of Baishakh, there was Rs. 5,000 in Ram's bank account. Rs. 30,000 was credited as a salary to his account on 7$^{th}$ Baisakh. He paid Rs. 945 as an electricity tariff from the account on 9$^{th}$ Baisakh. Rs. 500 is also recharged in his mobile phone from his account on 11$^{th}$ Baisakh. He paid Rs. 10,000 to the ABC food store from the account on 15$^{th}$ Baisakh Now find out how much money is left in his account.

4. The first integer is 2 more than five times of the second integer. If the first integer is 77, then find the second integer.

5. A tank of 1000 liter capacity has 500 liters of water. Two taps are fitted to it. In 1 minute, tap A fills 25 liters of water and tap B drains out 15 liters of water. How many liters of water are left in the tank if both the taps are opened for 6 minutes?

6. Write down five examples of integer using addition and subtraction in our daily life and present them in the class.

## Project Work

Write down five examples of integer using addition and subtraction in our daily life and present them in the class.

## Answer

| | | | |
|---|---|---|---|
| 1. (a) +2 | (b) –5 | (c) +36 | (d) –40 |
| 2. (a) 7 | (b) 4 | (c) blue house, 3 | |
| 3. Rs. 23555 | 4. 15 | 5. 560 *l* | |

## Lesson 4 Rational Number

### 4.0 Review

Take any two numbers from the set of integers Z = {…, –3, –2, –1, 0, 1, 2, 3, …}. Which operation would be possible of those numbers among addition, subtraction, multiplication and division? Discuss.

### 4.1 Introduction to Rational Number

#### Activity 1

Study the given condition and discuss the questions asked on the basis of number lines.

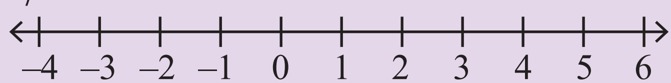
Two numbers 3 and –4 are taken from the set of integers. Among them addition, subtraction, multiplication and division operation have been done.

$3 + (-4) = -1$

$3 - (-4) = 3 + 4 = 7$

$3 \times (-4) = -12$

$3 \div (-4) = \frac{3}{-4}$




(a) Is the result always integer when two integers are added, subtracted and multiplied?

(b) What is the quotient between the integers?

(c) Is $\frac{3}{-4}$ in the number line of the integers?

Adding, subtracting and multiplying any two integers always gives the integer. But dividing one integer by another integer may not always be integer.

For example: $\frac{2}{3}, \frac{1}{2}, \frac{4}{5}$ are not integers. These come in the form of $\frac{a}{b}$. Such numbers are rational numbers. The set of rational numbers is denoted by Q.

$$Q = \{..., -4, -3, \frac{-5}{2}, -2, \frac{-3}{2}, -1, \frac{-1}{2}, 0, \frac{2}{3}, \frac{1}{2}, 1, ...\}$$

If any number can be expressed in the form of $\frac{a}{b}$ then such number is called rational number. Here $a$ and $b$ are integers and $b \neq 0$.

**Activity 2**

Discuss the following questions in the group with friends and present the conclusion in the class.

(a) Are all natural numbers rational numbers?

(b) Are all integers rational numbers?

Since all integers belong to the set of rational numbers, the set of integers is proper subset of the set of rational numbers. So it can be written as $Z \subset Q$.

## Properties of Rational Numbers

Discuss by observing the characteristics of the rational numbers given below.

### 1. Identity Property

| Identity property of addition | Identity property of multiplication |
|---|---|
| $\frac{1}{2} + 0 = 0 + \frac{1}{2} = \frac{1}{2}$ | $\frac{1}{2} \times 1 = 1 \times \frac{1}{2} = \frac{1}{2}$ |
| $\frac{-2}{3} + 0 = 0 + \frac{-2}{3} = \frac{-2}{3}$ | $\frac{-2}{3} \times 1 = 1 \times \frac{-2}{3} = \frac{-2}{3}$ |
| Adding zero (0) to any rational number gives the same number. It is called the identity property of addition. | Multiplying any rational number by 1 gives the same number It is called the identity property of multiplication. |

## 2. Inverse Property

| Inverse property of addition | Inverse property of multiplication |
|---|---|
| $-1 + 1 = 0$ | $2 \times \frac{1}{2} = 1$ |
| $\frac{-1}{2} + \frac{1}{2} = 0$ | $\frac{2}{3} \times \frac{3}{2} = 1$ |
| In any rational number $\frac{a}{b}$ adding $\frac{-a}{b}$ becomes zero, then it is called the inverse property of the addition. $\frac{a}{b}$ and $\frac{-a}{b}$ are inverse to each other. | If any rational number $\frac{a}{b}$ multiplied by $\frac{b}{a}$ becomes 1, then it is called inverse property of multiplication. $\frac{a}{b}$ and $\frac{b}{a}$ are considered to be the inverse to each other. |

## 3. Commutative Property

| Commutative property of addition | Commutative property of multiplication |
|---|---|
| $\frac{1}{2} + \frac{2}{3} = \frac{2}{3} + \frac{1}{2}$ | $\frac{1}{2} \times \frac{2}{3} = \frac{2}{3} \times \frac{1}{2}$ |
| If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers then $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ is called the commutative property of the addition. | If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers then $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$ is called the commutative property of the multiplication. |

## 4. Associative Property

| Associative property of addition | Associative property of multiplication |
|---|---|
| $\frac{1}{2} + \left(\frac{2}{3} + \frac{3}{5}\right) = \left(\frac{1}{2} + \frac{2}{3}\right) + \frac{3}{5}$ <br> If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are rational numbers then $\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$ is called the associative property of the addition. | $\frac{1}{2} \times \left(\frac{2}{3} \times \frac{3}{5}\right) = \left(\frac{1}{2} \times \frac{2}{3}\right) \times \frac{3}{5}$ <br> If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are rational numbers then $\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f}$ is called the associative property of the multiplication. |

## 5. Closure Property

| Closure property of addition | Closure property of multiplication |
|---|---|
| $\frac{1}{2}$ and $\frac{2}{3}$ are rational numbers. $\frac{1}{2}+\frac{2}{3}=\frac{3+4}{6}=\frac{7}{6}$ is also a rational number. | $\frac{1}{2}$ and $\frac{2}{3}$ are rational numbers. $\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$ is also a rational number. |
| The sum of two rational numbers is also a rational number. It is called closure property of addition. | The product of two rational numbers is also a rational number. It is called closure property of multiplication |

## 4.2 Decimal and Rational Number

**Activity 3**

Fill in the blanks by converting the fractions given below in the table into decimal numbers and discuss the findings.

| S.N. | Fractions | Decimal Numbers |
|---|---|---|
| 1 | $\frac{1}{2}$ | 0.5 |
| 2 | $\frac{1}{3}$ | 0.333 |
| 3 | $\frac{2}{7}$ | 0.285714285714 |
| 4 | $\frac{2}{3}$ | ... |
| 5 | $\frac{3}{10}$ | ... |
| 6 | $\frac{5}{13}$ | ... |

When converting fractions into decimals, it can be expressed as terminating, non-terminating and recurring decimals.

## 1. Terminating Decimal

Discuss by observing the example given below.

$\frac{1}{4} = 0.25$, $\frac{1}{8} = 0.125$, $\frac{1}{3} = 1.5$

Converting $\frac{1}{4}, \frac{3}{8}, \frac{3}{2}$ into the decimal number, the numbers after the decimal end in a certain place.

When the denominator of the rational number divides the numerator, after the decimal number terminates or ends in the quotient, then such a number is called terminating decimal numbers.

## 2. Non Terminating Recurring Decimal

Convert the fractions $\frac{1}{3}, \frac{2}{9}, \frac{4}{11}$ into decimal numbers. Discuss what the conclusion comes based on the results.

$\frac{1}{3} = 0.3333...$

$\frac{2}{9} = 0.2222...$

$\frac{4}{11} = 0.363636...$

$\frac{2}{7} = 0.285714285714...$

0.333…… can be written as 0.3 too.

Converting the above fractions into decimal numbers, the decimal parts does not terminate or end. The same digit or block of digits are repeated. Such a number is called non-terminating recurring decimal number.

Terminating decimal numbers and non-terminating decimals numbers are called rational numbers.

*Note: If the denominators of the rational numbers have a multiples of 2 or 5, then that numbers are terminating decimal number. For example,* $\frac{1}{2}, \frac{1}{5}, \frac{7}{10}, \frac{7}{25}, \ldots$

*If there are numbers other than 2 and 5 in the denominator of the rational number, then that numbers are non-terminating recurring decimal number. For example,* $\frac{1}{3}, \frac{2}{3}, \frac{5}{7}, \ldots$

## 4.3 Conversion of Decimal into Fraction

### I. Conversion of terminating decimal numbers into fraction

Discuss how 0.75 can be converted into the fraction.

0.75 means 75th part of hundredths.

So, $\frac{\cancel{75}^{3}}{\cancel{100}_{4}} = \frac{3}{4}$

> 0.75 has two numbers after the decimal. So, 0.75 has to be multiplied and divided by 100.

Another method: converting 0.75 into the fraction

$$\frac{0.75 \times 100}{100}$$

$$= \frac{75}{100}$$

$$= \frac{3}{4}$$

### II. Conversion of non-terminating recurring decimal numbers into fraction

Discuss how we can convert $0.\overline{3}$ into the fraction.

Let, $x = 0.\overline{3}$

$x = 0.33\ \ldots$ (i)

> There is same number 3 repeated after the decimal in $0.\overline{3}$, so it should be multiplied by 10.

Multiplying the equation (i) by 10, we get,

$10x = 3.33\ \ldots$ (ii)

Now, subtracting the equation (i) from the equation (ii),

$10x - x = 3.33\ldots - 0.33\ldots$

or, $9x = 3$

or, $x = \frac{3}{9}$

or, $x = \frac{1}{3}$

Therefore, $0.\overline{3} = \frac{1}{3}$

## Example 1

**Convert the following fraction into decimals and seperate terminating and non-terminating recurring decimal numbers.**

(a) $\frac{5}{8}$ (b) $\frac{2}{3}$

**Solution**

(a) $\frac{5}{8}$

$= 0.625$

0.625 is a terminating decimal number.

$$\begin{array}{r} 0.625 \\ 8\overline{\big)\ 50} \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

(b) $\frac{2}{3}$

$= 0.666...$

$= 0.\overline{6}...$

$0.\overline{6}$ is a non-terminating recurring decimal number.

$$\begin{array}{r} 0.666 \\ 3\overline{\big)\ 20} \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

## Example 2

**Convert the following decimal numbers into fraction.**

(a) 0.35 (b) $0.\overline{41}$

**Solution**

(a) 0.35

$$= \frac{0.35 \times 100}{100} = \frac{35}{100} = \frac{7}{20}$$

(b) $0.\overline{41}$

Let, $x = 0.\overline{41}$

$x = 0.4141...$ (i)

Multiplying the equation (i) by 100, we get,

$100x = 41.4141...$ (ii)

Now, subtracting the equation (i) from the equation (ii),

$100x - x = 41.4141... - 0.4141...$

or, $99x = 41$

or, $x = \frac{41}{99}$

Therefore, $0.\overline{41} = \frac{41}{99}$

### Example 3

**Find any two rational numbers lying between $\frac{1}{2}$ and $\frac{5}{6}$.**

**Solution**

To find any two rational numbers between $\frac{1}{2}$ and $\frac{5}{6}$

Any one number $= \dfrac{\frac{1}{2}+\frac{5}{6}}{2}$ (Finding the average value of $\frac{1}{2}$ and $\frac{5}{6}$

$= \frac{3+5}{6} \times \frac{1}{2} = \frac{5}{6} \times \frac{1}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{4}{6} = \frac{2}{3}$

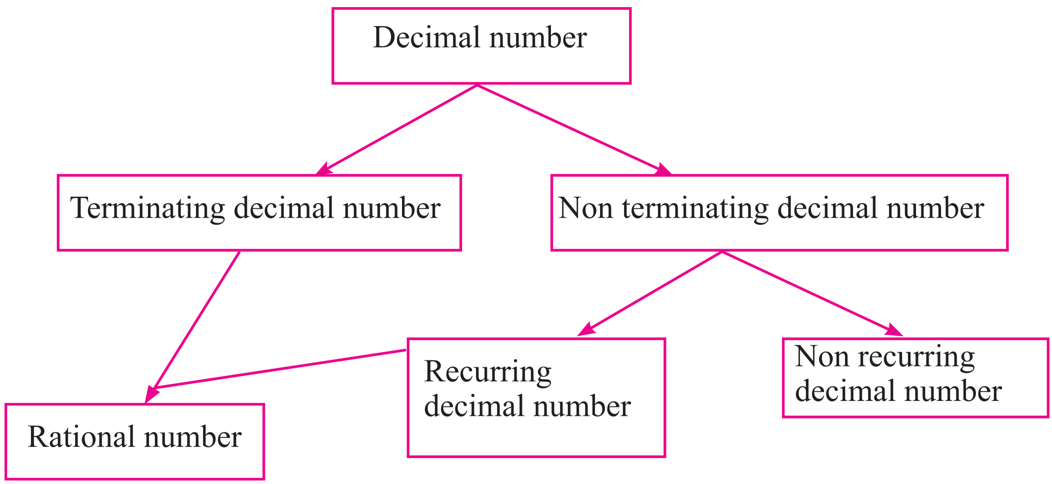
Another number = average value of $\frac{1}{2}$ and $\frac{2}{3}$

$= \dfrac{\frac{1}{2}+\frac{2}{3}}{2}$

$= \frac{3+4}{6} \times \frac{1}{2} = \frac{7}{12}$

Any two rational numbers between $\frac{1}{2}$ and $\frac{5}{6}$ are $\frac{2}{3}$ and $\frac{7}{12}$

The decimal number can be shown from the chart given below:




## Exercise 4.1

**1. Convert the following fraction into decimals and seperate terminating and non-terminating recurring decimal numbers:**

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{2}{7}$ (d) $\frac{15}{2}$ (e) $\frac{17}{13}$

(f) $\frac{55}{10}$ (g) $\frac{37}{20}$ (h) $\frac{25}{17}$ (i) $\frac{12}{25}$

**2. Write the additive inverse of the following numbers:**

(a) $\frac{2}{5}$ (b) $\frac{-5}{7}$ (c) $\frac{22}{12}$ (d) $\frac{12}{7}$ (e) $\frac{-11}{8}$

**3. Write the multiplicative inverse of the following numbers:**

(a) $\frac{3}{4}$ (b) $\frac{25}{10}$ (c) $\frac{-2}{3}$ (d) $\frac{22}{12}$ (e) $\frac{1}{8}$

**4. Convert the following decimal numbers into fraction:**

(a) 0.5 (b) $0.\overline{7}$ (c) $0.\overline{24}$ (d) $0.\overline{27}$

(e) $1.\overline{57}$ (f) 2.35 (g) 7.025

5. Write the any two rational numbers lying between $\frac{1}{2}$ and $\frac{3}{4}$.

6. Find any two rational numbers lying between $\frac{1}{3}$ and $\frac{3}{4}$.

**7. Write the answers of the following questions with reasons:**

(a) Are all the natural numbers rational numbers?

(b) Are all the whole numbers rational numbers?

(c) Are all the integers rational numbers?

(d) Is zero (0) a rational number?

(e) Are all the rational numbers integers?

## Project Work

Show the relationship among natural numbers, whole numbers, integers and rational numbers in the figure and present them in the class.

## Answer

Show answers to your teachers.

# Lesson 5 Fraction and Decimal

## 5.1 Fraction

### 5.1.0 Review

Discuss in a group and calculate the problem related to fraction given below:

(a) $\frac{1}{5} + \frac{1}{7}$ (b) $5\frac{5}{6} - 1\frac{2}{3}$ (c) $6 \times \frac{2}{5}$ (d) $12 \div \frac{2}{5}$

### 5.1.1 Simplification of Fractions

**Activity 1**

Study the following mathematical problem and discuss the questions asked:

The monthly income of Sirjana is Rs. 24,000. She spent one-third of her monthly income on education. Similarly, if she spent half of income on food,

(a) What part of her total income did she spend?

(b) How much did she spend in total?

Here, one-third of the income spent on education = one part out of three part (one third) = $\frac{1}{3}$

Half of the income spent on food = one part out of 2 part (half) = $\frac{1}{2}$

Total expenditure = ?

To find out the total expenditure, both expenditure must be added.

So, total expenditure = $\frac{1}{3} + \frac{1}{2}$

$= \frac{1}{3} \times \frac{2}{2} + \frac{1}{2} \times \frac{3}{3}$

Making equal denominators to unequal denominators

$= \frac{2}{6} + \frac{3}{6}$

Total expenditure = $\frac{5}{6}$ (writing in lowest term)

Now, Total expenditure amount = $\frac{5}{6}$ of total income.

$$= \frac{5}{6} \times \text{Rs. } 24000$$

$$= 5 \times \text{Rs. } 4000$$

$$= \text{Rs. } 20,000$$

## Another Method (by Model Drawing)

Since one-third of the income is spent on education, dividing her income into three equal parts,

| Sirjana's income = Rs. 24000 | | |
|---|---|---|
| $x$ | $x$ | $x$ |

$3x = 24000$

or, $x = \frac{24000}{3}$

or, $x = 8000$

Hence, expenditure on education = Rs. 8000

Again, half of the income is spent on food,

| Sirjana's income = Rs. 24000 | |
|---|---|
| $y$ | $y$ |

$2y = 24000$

or, $y = \frac{24000}{2}$

or, $y = 12000$

Hence, expenditure on food = Rs. 12000

Therefore, total expenditure = expenditure on education + expenditure on food

$$= 8000 + 12000$$

$$= \text{Rs. } 20,000$$

Part of total expenditure = $\frac{\text{total expenditure}}{\text{total income}} = \frac{\text{Rs. } 20000}{\text{Rs. } 24000} = \frac{5}{6}$

## Example 1

**There are 48 students in class 7 of Shanti Niketan School. Two-thirds ($\frac{2}{3}$) of them are boys and the rest are girls,**

(a) What is the number of girls?

(b) What is the number of boys? Find out.

### Solution

Here, total number of students = 48

Number of girls = $\frac{2}{3}$ part of total students

$= \frac{2}{3} \times 48$

$= 32$

The number of boys = Total number of students - number of girls

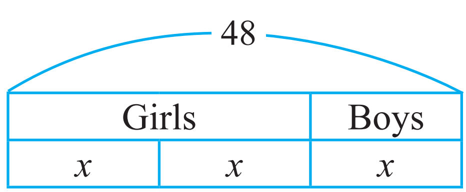
$= 48 - 32$

$= 16$

Hence, there are 32 girls and 16 boys.

### Another Method (by Model Drawing)

The number of girls is two-thirds,




$3x = 48$

$x = \frac{48}{3}$

$\therefore x = 16$

Girls = 2 part

Boys = 1 part

Numbers of girls = 2 part

$= 2 \times 16$

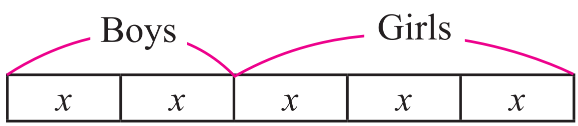
$= 32$

Numbers of boys = 48 – 32 = 16

## Example 2

**In a basic school, $\frac{2}{5}$ part of the total students are boys. If there are 90 girls, how many boys are there? How many students are there in total?**

**Solution**




Here, $3x = 90$

Or, $x = \frac{90}{3}$

Or, $= 30$

Numbers of boys $2x = 30 \times 2$

$= 60$

Total numbers of students $= 5x = 5 \times 30 = 150$

## Example 3

**Prabin bought a cake on his birthday. Among his friends, Kripa ate $\frac{1}{2}$ portion, Aman ate $\frac{1}{3}$ portion and Sandeep ate $\frac{1}{6}$ portion. Find out who ate the most portion of cakes.**

**Solution**

Portion of the cake eaten by Kripa $= \frac{1}{2}$

Portion of the cake eaten by Aman $= \frac{1}{3}$

Portion of the cake eaten by sandeep $= \frac{1}{6}$

Now, find the LCM of the denominator of the three fractions.

LCM = 6

Multiples of 2 = 2, 4, 6, 8, …
Multiples of 3 = 3, 6, 9, …
Multiples of 6 = 6, 12, …
Lowest common multiples = 6

Now, make the denominator 6 of all fractions.

$$\frac{1}{2} = \frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$

$$\frac{1}{3} = \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

$$\frac{1}{6} = \frac{1}{6} \times \frac{1}{1} = \frac{1}{6}$$

Here3 times of $\frac{1}{6}$ is $\frac{3}{6}$, 2 times of $\frac{1}{6}$ is $\frac{1}{6}$

So, $\frac{3}{6}$ is the greatest fraction among the 3 fractions.

$\therefore$ Kripa ate $\frac{1}{2}$ portion of the cake that is the most.

## Exercise 5.1

**1. Simplify:**

(a) $\left(11\frac{1}{13} - 1\frac{3}{8}\right) + 2\frac{7}{8}$ (b) $13\frac{1}{9} - \left(4\frac{8}{9} - 6\frac{5}{11}\right)$

(c) $\frac{1}{2} + \frac{1}{3} \times \frac{6}{2} - \frac{4}{5} \div \frac{1}{2}$ (d) $\left(\frac{1}{2} + \frac{2}{3}\right) \div \frac{3}{5} \times \frac{4}{5} + \frac{5}{9} - 2$

2. Subtract $\left(6\frac{1}{8} - 8\right)$ from 30 and add $1\frac{1}{4}$

**3. Sandeep's monthly income is Rs 27,000. He has spent $\frac{1}{5}$ part of his income on food. If $\frac{1}{10}$ part spent on clothing and $\frac{2}{5}$ part spent on transportation, then:**

(a) How many parts did he spend in total?

(b) How many parts did he save?

(c) Find out how much money he has saved.

4. At the fun fair, Rema spent $\frac{1}{5}$ of her money on entertainment and $\frac{1}{2}$ on food. Find out in which title she has spent the most amount.

5. **Out of 40 students in a class, $\frac{1}{5}$ students like English subject. $\frac{2}{5}$ Students like mathematics. The rest of the students like science subjects.**

(a) Find the number of students who like English.

(b) Find the number of students who like Mathematics.

(c) Write in the form of fraction to the students who like science.

6. **Ranju's father gave her Rs 60,000. She bought books for one-third of her money. She bought clothes with one-fourth of the rupees. One-fifth of the rupees is spent on her travel.**

(a) How much did she spend?

(b) How much money had been spent in total?

(c) Find out how much money she saved.

7. **Ritu ate $\frac{3}{5}$ parts of an apple and the rest was eaten by his brother, Suman.**

(a) Write in fraction the part of the apple that Suman ate.

(b) Find out who ate the most part of apple.

8. **Mohan gave $\frac{2}{3}$ parts of his money to his wife. He gave $\frac{1}{5}$ part of the remaining amount to his son and $\frac{1}{3}$ parts to his daughter. If he gave Rs. 60,000 to his wife then,**

(a) How much money did he give to his son?

(b) How much money did he give to his daughter?

(c) How much more money was given between son and daughter?

9. Prakriti has spent $\frac{1}{4}$ parts of the money. She has lost $\frac{2}{5}$ parts from the remaining amount. If the lost money is Rs. 900, find out how much money he had at the beginning.

10. There are $\frac{4}{5}$ part girls in one school. The school's picnic program includes $\frac{1}{4}$ part girls and $\frac{1}{3}$ part boys. If 190 students went to the picnic program, find out the total number of students in the school

## Project Work

How much time do you spend on the following activities out of the time you spend in a day at school? Present in the class by writing the form of the fraction.

(a) Morning prayer
(b) Lunch time
(c) Reading time
(d) Time for other activities

## Answer

1. (a) $12\frac{15}{26}$ (b) $14\frac{67}{99}$ (c) $\frac{-5}{2}$ (d) $\frac{1}{9}$

2. $33\frac{1}{8}$

3. (a) $\frac{7}{10}$ part (b) $\frac{3}{10}$ part (c) Rs. 8,100

4. Show answers to your teachers.

5. (a) 8 (b) 16 (c) $\frac{2}{5}$ part

6. (a) $\frac{47}{60}$ part (b) Rs. 4,700 (c) Rs. 1300

7. (a) $\frac{2}{5}$ (b) Ritu $\frac{1}{5}$

8. (a) Rs. 6000 (b) Rs. 10,000 (c) to daughter: Rs. 4000

9. Rs. 3000

10. 600

## 5.2 Decimal

### 5.2.0 Review

Calculate of the decimal number given below:

(a) $8.97 + 23.2$ (b) $3.6 \times 5.8$

(c) $7.7 - 2.8$ (d) $17.40 \div 4$

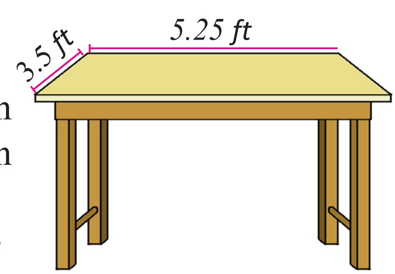
What should be considered when adding, subtracting, multiplying and dividing of decimal number? Discuss in pairs and find out the conclusion.

### 5.2.1 Simplification of Decimal

**Activity 1**

Isha was making some patterns to decorate the classroom. It requires 4. 5 cm length squared paper. He has a rectangular paper with 54 cm length and 4.5 cm breadth. How many pieces can be made having length 4.5 cm from that paper? She divided 54 by 4.5 to find the pieces of paper. Is this right? Discuss and conclude it.

**Activity 2**




The rectangular surface of the table shown in the picture has the length 5.25 ft and breadth 3.5 ft.

(a) What is the perimeter of the table?

(b) Which mathematical operation should be used to find the perimeter? Discuss.

Here to find the perimeter of the table,

Both the length and the breadth of the table should be added twice.

Or, perimeter of the table $= 2(l + b)$

So, perimeter of the table $= 2(5.25 \text{ ft} + 3.5 \text{ ft})$

$= 2 \times 8.75 \text{ ft}$

$= 17.5 \text{ ft}$

## Example 1

**How can the following mathematical problem be simplified? Conclude by discussing the questions asked.**

**Simplify:**

$7.5 + \{6.72 \div 2.8\ (3.59 - 1.49)\}$

(a) Is it possible to divide 6.72 by 2.8 without subtracting 1.49 from 3.59?

(b) Which operation should be done within brackets at first?

(c) What is the order in brackets when simplifying?

When simplifying, the operation of division, multiplication, addition and subtraction is done respectively, but here, first of all, we have to subtract 1.49 from 3.59. The result should be multiplied by 2.8, then we should divide to 6.72. Therefore (3.95 – 1.49) is placed in the small bracket and {6.72÷ 2.8 (3.59–1.49)} is placed in the middle bracket.

**Solution**

Here $7.5 + \{6.72 \div 2.8\ (3.59 - 1.49)\}$

$= 7.5 + \{6.72 \div 2.8\ (2.1)\}$

$= 7.5 + \{6.72 \div 5.88\}$

$= 7.5 + 1.14$

$= 8.64$

Simplifying the decimals with four mathematical operations (+, – , × , ÷) and brackets, as in the simplification of whole numbers, should be done after doing the rest of the operations, included in the brackets at first. The brackets used in simplification should be followed by operations consisting of small bracket ( ), middle bracket {} and large bracket [ ] respectively.

## Exercise 5.2

1. **Simplify:**

   (a) $1.44 \div 1.2 + 6.2$

   (b) $12.75 - \{4.38 - (2.4 \times 4.32 \div 3.6 - 0.85)\}$

   (c) $\dfrac{1.2 \times 1.2 - 0.4 \times 0.4}{2.4 - 1.6}$

   (d) $\dfrac{4.5 \times 4.5 - 2.1 \times 2.1}{4.5 + 2.1}$

2. Area of the triangle = $\frac{1}{2}$ × base × height. If the base of a triangle is 25.75 cm and the height is 30.15 cm then find the area.

3. The length of a rectangular garden is 22.66 cm, and breadth is 15.65 cm.

   (a) What is the area of the garden?

   (b) Find out the perimeter of the garden.

4. The perimeter of a squared table is 24.4 ft. Find the length of the table.

5. The area of a rectangular field is 215.66 $m^2$. If the field is 67.35 m long, find out how wide it is.

6. If the length of a squared field is 8.45 m, then find the perimeter of that field.

## Project Work

Search today's exchange rates of currency from magazines or the internet. Note down the buying and selling rates of currencies of any five countries and find the differences among them and present it in the class.

## Answer

| | | | |
|---|---|---|---|
| 1. (a) 7.4 | (b) 10.4 | (c) 1.6 | (d) 2.4 |
| 2. 388.18 $cm^2$ | 3. (a) 354.62 $m^2$ | (b) 76.62 m | |
| 4. 6.1 *ft* | 5. 3.20 m | 6. 33.8 m | |

## Lesson 6

# Ratio and Proportion

## 6.1 Ratio

### 6.1.0 Review

The weight of Anuja is 30 kg and weight of Ramesh is 60 kg. Discuss how the weight of Anuja and Ramesh can be compared.

As difference,

Difference between weight of Anuja and Ramesh = (60–30) kg = 30 kg

As quotient,

$$\frac{\text{Weight of Anuja}}{\text{Weight of Ramesh}} = \frac{30\text{ kg}}{60\text{ kg}} = \frac{1}{2}$$

*The difference itself refers to how many digits are small or large and quotient refers to how many times small or large.*

### 6.1.1 Introduction to Ratio

**Activity 1**

Observe the given table and discuss the questions asked:

| Item | Price |
|---|---|
|  | Rs. 10 |
|   | Rs. 70 |
|  | Rs. 5 |

(a) How many times is the price of copy than the price of pencil?

(b) How many times is the price of eraser than the price of pencil?

$$\frac{\text{price of copy}}{\text{price of pencil}} = \frac{\text{Rs.}70}{\text{Rs. }10} = \frac{7}{1}$$

The price of copy is 7 times the price of pencil.

$$\frac{\text{price of eraser}}{\text{price of pencil}} = \frac{\text{Rs. }5}{\text{Rs. }10} = \frac{1}{2}$$

The price of eraser is half the price of pencil.

Ratio is the comparison on the basis of one quantity to another quantity.

The ratio having the same unit of two quantity $a$ and $b$ is $\frac{a}{b}$ or $a$: $b$, where, $b \neq 0$. $a$:$b$ can be read as $a$ is to $b$ where $a$ and $b$ are the terms of ratio.

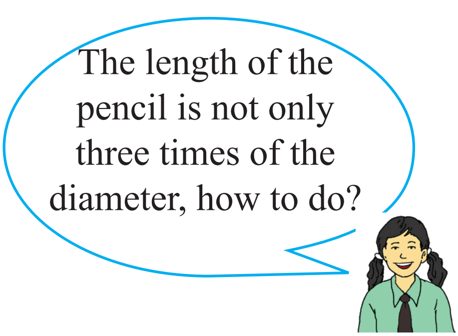
## Simplest form of a Ratio

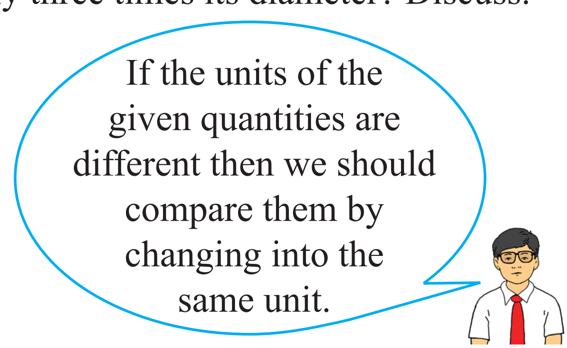
### Activity 2

The length of the pencil shown below is 18 cm and diameter is 6 mm. What is the ratio of the diameter of a pencil to its length?


Can be written as $\frac{18\text{ cm}}{6\text{ mm}} = \frac{3}{1}$ ?

Is the length of the pencil only three times its diameter? Discuss.



Changing the length of the pencil in millimeter,

$18 \text{ cm} = 18 \times 10 \text{ mm} = 180 \text{ mm}$

So, ratio of the diameter of pencil and its length can be written as

$$= \frac{10 \text{ mm}}{180 \text{ mm}} = \frac{1}{18}$$

- Ratio is the comparison of two quantities having the same unit.
- Ratio has no unit.

## Relation of Ratio and Fraction

### Activity 3

Draw a circle. Divide it into five equal parts.

(a) Write in fraction to the shaded part.

(b) Find the ratio of the shaded part and non-shaded part.

(c) Discuss what are the differences between fraction and ratio.

Here,

Here, fraction of the shaded part $= \frac{3}{5}$

The ratio of shaded and non-shaded part $= \frac{2}{3} = 2:3$

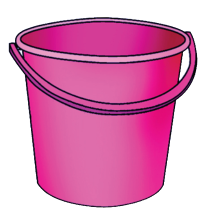
In fraction $\frac{3}{5}$ refers to 3 parts out of 5 parts.

In the ratio 2:3 refers in total there are $2 + 3 = 5$ parts.

### Example 1

**Write the ratio of the capacity of the two buckets shown in the figure.**

Capacity: 9 *l*



Capacity: 15 *l*

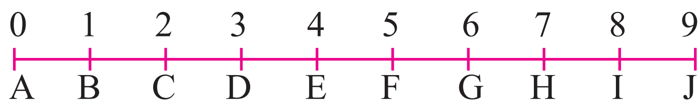
**Solution**

Here $\frac{\text{capacity of the first bucket}}{\text{capacity of the second bucket}} = \frac{9\,l}{15\,l} = \frac{3 \times 3\,l}{3 \times 5\,l} = \frac{3}{5} = 3:5$

Therefore, the capacity of first bucket: the capacity of second bucket = 3:5

## Example 2

**Each part represents 1 cm in the number line given below.**




**What is the ratio of AC and AF?**

**Solution**

Here AC : AF

$= \frac{AC}{AF}$

$= \frac{2\text{ cm}}{5\text{ cm}} = \frac{2}{5} = 2:5$

## Example 3

**Bishnu obtained 75 marks out of 100 full marks in mathematics. In science, he obtained 50 marks out of 75 full marks. In which subject did he obtain better marks?**

**Solution**

Here, fraction of marks obtained in mathematics $= \frac{75}{100} = \frac{3}{4}$

Fraction of marks obtained in science $= \frac{50}{75} = \frac{2}{3}$

Making equal to the denominator,

Fraction of marks obtained in mathematics $= \frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$

Fraction of marks obtained in mathematics $= \frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$

$\therefore \frac{9}{12} > \frac{8}{12}$ So, Bishnu obtained better marks in mathematics.

## Example 4

**The height of Anju and Aman are 145 cm and 165 cm respectively. Find the ratio of their height.**

**Solution**

Here, the ratio of the height of Anju and Aman = $\frac{145 \text{ cm}}{165 \text{ cm}}$

$= \frac{5 \times 29}{5 \times 33} = \frac{39}{33} = 29 : 33$

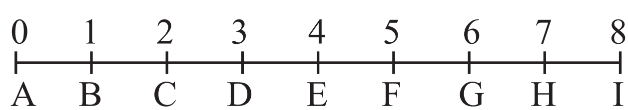
## Exercise 6.1

**1. Write the ratio of each of the following and convert it to the lowest term:**

(a) 10 cm and 100 cm (b) Rs. 180 and Rs. 240 (c) 10 kg and 2 kg

(d) 8 hours and 24 hours (e) 250 *ml* and1000 *ml* (e) 2.5 kg and 7.5 kg

**2. Each part represents 1 cm in the number line given below. Find the ratio of the distance asked:**

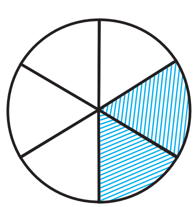



(a) AB : AG (b) AC : DI (c) CF : CH

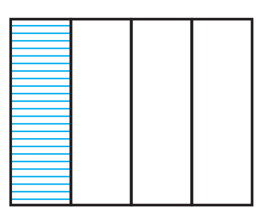
(d) BG : BI (e) AF : AI

**3. Find the ratio of the shaded part to non-shaded part from the given figure:**

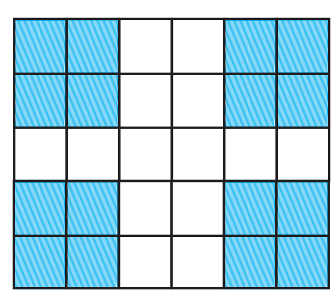
(a)



(b)



(c)



**4. There are 450 students in a school, among them 180 are girls then,**

(a) find the ratio of girls and total students.
(b) find the ratio of boys and total students.
(c) find the ratio of girls and boys.

5. There are 25 teachers and 500 students in a school, then find the ratio of teachers and students.

**6. The height of Pramesh is 165 cm and the height of Pramila is 150 cm then,**

(a) find the ratio of the height of Pramesh and Pramila.
(b) find the ratio of the height of Pramila and Pramesh.

**7. Rahul obtained 40 marks out of 50 full marks in English, 20 marks out of 30 full marks in Nepali and 13 marks out of 20 full marks in Mathematics.**

(a) Which result is better in the subjects English and Nepali?
(b) Which result is better in the subjects Nepali and Mathematics?
(c) Which is the best result in these subjects? Compare by finding the ratio.

8. Find out how much money will Alisa and Dipesh get while dividing Rs 60,000 in the ratio of 5:4.

## Project Work

Note down the number of boys and girls of grade 5 to 10 in your school. With class-wise for each class,

(a) Find the ratio of girls and boys.
(b) Find the ratio of girls and total students.
(c) Find the ratio of boys and total students and present the conclusion in the classroom,

## Answer

1-3 Show the answers to your teacher.

4. (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{3}$ 5. 1:20 6. (a) 11:10 (b) 10:11

7. Show the answers to your teacher. 8. Rs. 2500, Rs. 3500

## 6.2 Proportion

### 6.2.0 Review

Complete the given table so that there are equivalent fractions of $\frac{2}{5}$.

| | |
|---|---|
| $\frac{2}{5} = \frac{4}{10}$ | $\frac{2}{5} = \frac{10}{\quad}$ |
| $\frac{2}{5} = \frac{6}{\quad}$ | $\frac{2}{5} = \frac{\quad}{30}$ |
| $\frac{2}{5} = \frac{\quad}{20}$ | $\frac{2}{5} = \frac{14}{\quad}$ |

Are all fractions $\frac{4}{10}$, $\frac{6}{15}$, $\frac{8}{10}$ equivalent? Conclude after discussion.

### 6.2.1 Introduction to Proportion

**Activity 1**

Take two sheets of squared paper of equal size. Divide the first in four equal parts and the second into 16 equal parts.

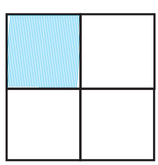


Fig. I

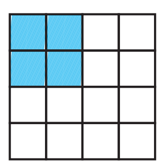


Fig. II

Find the ratio of the shaded part and non-shaded part from the Fig. I.

$$\frac{\text{shaded part}}{\text{non-shaded part}} = \frac{1}{3}$$

Now find the ratio of the shaded part and non-shaded part from the Fig. II

$$\frac{\text{shaded part}}{\text{non-shaded part}} = \frac{4}{12} = \frac{1}{3}$$

So, $\frac{1}{3}$ and $\frac{4}{12}$ are equal ratios.

Equal ratios are called proportion.

## Activity 2

Dipendra obtained 25 marks out of 30 in the examination of mathematics. Narmada obtained 20 marks in the examination out of 24 full marks. Whose result is better? What is the ratio of their score? Discuss how it can be compared.

Ratio of obtained marks and full marks of Dipendra $= \frac{25}{30} = \frac{5}{6}$

Ratio of obtained marks and full marks of Narmada $= \frac{20}{24} = \frac{5}{6}$

Since, the ratio of $\frac{25}{30}$ and $\frac{20}{24}$ are equal, the result of both seems to be equal. So, here $\frac{25}{30} = \frac{20}{24}$

Since the two ratios are equal, these ratios are called proportions. Here, 25 is called the first term, 30 the second term, 20 the third term and 24 is called the fourth term.

Among the four numbers *a, b, c* and *d,* if the ratio of *a* and *b* and the ratio of *c* and *d* are equal then, *a, b, c* and *d* are said to be in proportion. It is written as $a : b :: c : d$ or $a \times d = b \times c$.

In the above example $\frac{25}{30}$ and $\frac{20}{24}$ are proportional.

Extremes

It is also written as 25 : 30 :: 20 : 24

Means

The two outer terms are called extremes. The two inner terms are called means. Here, 25 and 24 are extremes and 30 and 20 are means.

$25 \times 24 = 30 \times 20 = 600$

*The product of extremes and product of means is equal separately.*

## Example 1

### Are these numbers 3, 4, 9 and 12 in proportion?

**Solution**

Here, product of extremes = $3 \times 12 = 36$

Product of means = $4 \times 9 = 36$

Since product of extremes and product of means are equal, 3, 4, 9 and 12 are in proportion.

**Another method**

Ratio of the first and second numbers $= \frac{3}{4}$

Ratio of the third and fourth numbers $= \frac{9}{12} = \frac{3}{4}$

Since both the ratios are equal, 3, 4, 9 and 12 are in propertion.

## Exercise 6.2

**1. Which of the following ratios are equal? Write with reason:**

(a) $\frac{2}{3}$ and $\frac{4}{5}$ (b) $\frac{8}{4}$ and $\frac{2}{1}$

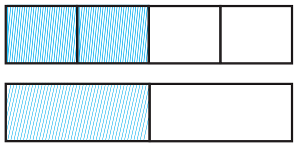
(c) $\frac{4}{5}$ and $\frac{12}{20}$ (d) $\frac{3}{5}$ and $\frac{9}{15}$

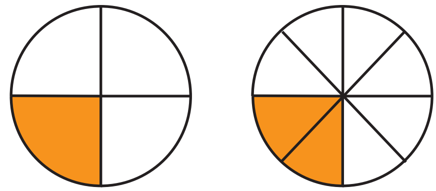
**2. Check whether the numbers given below are in proportion or not.**

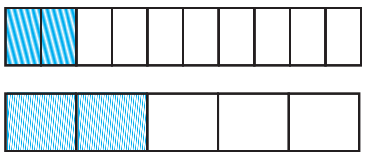
(a) 2, 3, 12, 20 (b) 7, 8, 14, 20

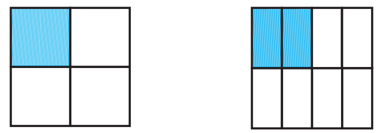
(c) 5 m, 3 m, 25 m, 35 m (d) 3 ft, 8 ft, 12 ft, 32 ft

3. **Write the ratio of shaded part and non-shaded part from the pair of figure given below. Write with reason if these figures are in proportion or not.**

(a) 

(b) 

(c) 

(d) 

4. **Tick (√ ) for the right answers and cross (×) for the wrong ones.**

(a) 16 : 24 :: 20 : 30

(b) 8 : 9 :: 24 : 27

(c) 32 m : 64 m = 6 sec : 12 sec

(d) 45 km : 60 km = 12 hours : 15 hours

5. **Write extremes and means separately from the proportions given below.**

(a) 40 : 200 :: 15 : 75

(b) 3 : 7 :: 21 : 49

(c) 10 : 55 :: 2 : 11

(d) 25 : 15 = 5 : 3

6. The ratio of the score of Anita is 10 : 12 in Mathematics and Science. If her score of mathematics is 80, then what is her score in science?

7. If the price of 6 kg oranges is Rs. 540, then find out the price of 8 kg oranges.

8. A restaurant made lemonade by putting 200 ml of honey in 3 liters of hot water for 15 people. Later 3 people were added. How many ml of honey should be mixed in 600 ml of water to make the same quality of lemonade?

## Project Work

Take two sheets of squared paper, divide them into different parts so that the ratio of these papers are equal and present them in the class.

## Answer

1-5. Show the answers to your teacher. 6. 96

7. Rs. 720 8. 40 *ml*

# Lesson 7 Profit and Loss

## 7.0 Review

Discuss the following questions:

(a) Subas bought a bag for Rs. 750. How much actual profit or loss did he get if he sold it at Rs. 800?

(b) Kamala bought a bag for Rs. 1000. After seeing minor defect in the bag, she sold it at Rs. 900. How much actual profit or loss did Kamala get?

Subas has bought the bag at low prices and sold them at high prices. So there was actual profit. Similarly, Kamala bought at high price and sold at low price. So there was loss.

Actual profit is the difference between selling price and cost price.

Profit = selling price – cost price
Loss = cost price – selling price

## 7.1 Problems of Profit and Loss with Percentage

### Activity 1

Study and discuss the following condition:

Mankaji is a businessman. He bought each t-shirt from a wholesaler at the rate of Rs 1000. The first t-shirt was sold at Rs. 1200. Similarly, after seeing it slightly torn in the second t-shirt, he sewed it and sold at Rs. 950.

(a) How much profit was made on the first T-shirt?

(b) What was loss on the second T-shirt?

(c) How can we find the actual profit on the first T-shirt in percentage?

(d) What was the loss percentage on the second T-shirt? How can it be found?

| In the first t-shirt, | In the second t-shirt, |
|---|---|
| Cost price = Rs. 1000 | Cost price = Rs. 1000 |
| Selling price = Rs. 1200 | Selling price = Rs. 950 |
| Actual Profit = Rs. 1200 – Rs. 1000 | Loss = Rs. 1000 – Rs. 950 |
| = Rs. 200 | = Rs. 50 |
| Profit x = $\frac{\text{actual profit}}{\text{CP}} \times 100\%$ | Loss $x$ = $\frac{\text{actual loss}}{\text{CP}} \times 100\%$ |
| Now, $\frac{\text{Rs. }200}{\text{Rs. }1000} \times 100\%$ | Now, $\frac{\text{Rs. }50}{\text{Rs. }1000} \times 100\%$ |
| = 20% (It is actual profit percent.) | = 5% (It is loss percent.) |

Profit percentage = $\frac{\text{actual profit}}{\text{CP}} \times 100\%$ → $\frac{\text{SP} - \text{CP}}{\text{CP}} \times 100\%$

Loss percentage = $\frac{\text{actual loss}}{\text{CP}} \times 100\%$ → $\frac{\text{CP} - \text{SP}}{\text{CP}} \times 100\%$

## Example 1

**Rajesh has bought a watch for Rs. 1200 and he sold at Rs. 1500. What would be the profit or loss? Find in percentage.**

### Solution

Here, cost price of the watch = Rs. 1200

Selling price of the watch = Rs. 1500

The selling price is more than the cost price, so there was profit.

Now, actual profit = selling price – cost price

= Rs.1500 – Rs. 1200

= Rs. 300

Actual profit percentage = $\frac{\text{actual profit}}{\text{CP}} \times 100\%$

$= \frac{\text{Rs.}300}{\text{Rs.}1200} \times 100\%$

= 25%

∴ Rajesh got 25% actual profit by selling this watch.

## Example 2

**Jyoti decided to sell an electric heater at 10 percent profit which was bought for Rs 2,500. How much should she sell the heater?**

### Solution

Here, cost price of the heater = Rs. 2500

Actual profit percentage = 10%

Selling price of the heater = ?

Adding 10% actual profit in the cost price Rs. 2500 of the heater is the selling price.

Actual profit = 10% of Rs. 2500

$= \frac{10}{100} \times \text{Rs. } 2500$

= Rs. 250

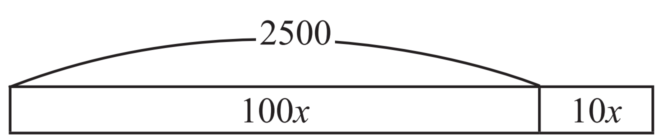
Now, selling price = cost price + profit

= Rs.2500 + Rs. 250

= Rs. 2750

Therefore, Jyoti has to sell the heater at Rs. 2750 to earn 10 % profit.

### Next method (By model drawing)




$100x$ = Rs. 2500

$x = 25$

Now, $110x = 25 \times 110$

= Rs. 2750

## Example 3

**By selling a mobile for Rs 21,000, a mobile seller gained 5%.**

(a) What is the cost price of the mobile?

(b) At what price should the mobile be sold to receive 10 % profit?

### Solution

Here, selling price of the mobile= Rs. 21000

Profit percentage = 5%

Cost price of the mobile= ?

(a) Let, cost price of the mobile = Rs. $x$

Now, selling price = cost price + profit

So, Rs. 21,000 = $x$ + 5% of $x$

or, Rs. 21000 = $x + \frac{x \times 5}{100}$

or, Rs. 21000 = $\frac{100x + 5x}{100}$

or, Rs. 21000 = $\frac{105x}{100}$

or, $105x$ = Rs. $21000 \times 100$

or, $x$ = Rs. $\frac{21000 \times 100}{105}$

or, $x$ = Rs. 20,000

So the mobile was bought for Rs 20,000.

(b) Calculating the selling price to receive 10% profit,

Now, selling price = cost price + profit

= Rs. 20,000 + Rs. 20,000 of 10%

= Rs. 20,000 + Rs. 20,000 $\times \frac{10}{100}$

= Rs. 20,000 + Rs. 2,000

= Rs. 22,000

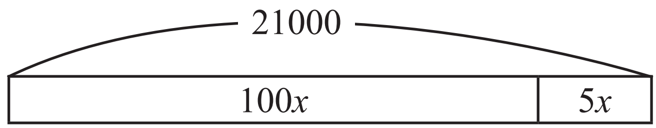
So to receive 10 % profit, the mobile has to be sold for Rs 22,000.

## Next method (By model drawing)

Here,

$105x = \text{Rs. } 21{,}000$

$x = 200$




Now, CP = $100x$ = Rs. 20,000

SP = $110x$

$110x = 110 \times 200$

$= \text{Rs. } 22{,}000$

## Exercise 7.1

1. What is the profit or loss and their percentage in the following condition? Find out.

| Cost price (Rs.) | Selling price (Rs.) |
|---|---|
| 300 | 350 |
| 550 | 500 |
| 7000 | 7720 |
| 5000 | 2750 |
| 12000 | 15000 |

2. Ajaya bought a television for Rs 30,500 and sold it for Rs 29,000. Find out what his loss perecentage is.

3. Find out what is the cost price of a sari if it is sold for Rs. 30,500 by gaining 10% profit.

4. What will be the selling price of a mobile bought at Rs. 30,000 by gaining 10% profit?

5. A fruit seller bought 10 kg oranges at the rate of Rs. 55 per kg. If he sold all the oranges at the rate of Rs. 60 per kg, what would be the profit percentage?

6. Sukman bought 50 bulbs at the rate of Rs 40 to each bulb. When he opened the packet of bulbs, 15 bulbs were broken. What would be the profit or loss? Find it in percentage.

7. When a book was sold to a bookseller for Rs. 575, the profit was 15 %. How much money was paid for the book? Find out the actual profit amount.

8. Rajendra bought a refrigerator for Rs 32,500. He spent Rs. 500 to bring the goods. Find out the profit percentage if he sold the refrigerator for Rs 33,500.

9. Aaitman bought the old house for Rs. 95,00,000. He spent Rs. 200,000 for house repairing. If the house was sold for Rs. 1,10,00,000, what would be the profit or loss, find in percentage.

10. Basmati bought 20 dozen bananas at the rate of Rs. 120 per dozen. Among them, 3 dozen bananas were spoiled. Find out at what rate the rest of the bananas should be sold to get 10 % profit.

## Project Work

Note down the price of any five items from two different shops. What is the percentage of profit you got from which shop? Present the findings in the classroom.

## Answer

1. (a) Profit: 16.66% (b) Loss: 9.09% (c) Profit: 10.28%
   (d) Loss 45% (e) Profit 25%
2. Loss: 4.91%
3. Rs. 3,000
4. Rs. 33,000
5. 9.09%
6. Profit: 5%
7. Rs. 500, Rs. 75
8. 1.51%
9. Profit: 13.40%
10. Rs. 155.30 per dozen.

# Lesson 8 Unitary Method

## 8.0 Review

Laxmi bought 30 bananas for Rs. 225 and brought them home. These bananas are not enough for the guests who have come to the house. So, 12 bananas have to be added again.

(a) How much money does she need to buy 12 bananas now?

(b) What should be done to find out the price of 1 banana?

(c) Discuss how to find out the price of 12 bananas.

The rule of computing the value of the same number of objects on the basis of given value of one unit of object and finding the value of a object from the values of the same number of objects is called unitary method.

## 8.1 Direct Variation

**Activity 1**

The number of copies and their price are given in the table below, fill in the prices based on the given price in the table below:

| Number of copies | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Total price (Rs). | 50 | 100 | 150 | | | |

Discuss the relationship between the number of copies and the price.

From the above table, as the number of copies increases, the price of copy is also increasing and as the number of copies decreases, the price of copy is also decreasing.Therefore, there is direct variation between the number of copies and the price.

*When one of the two quantities decreases or increases, the other quantity decreases or increases in the same proportion, then the relation of those quantities is called direct variation.*

## Example 1

**If the price of 3 copies is Rs. 270, then what is the price of 5 copies?**



### Solution

Here, the price of 3 copies = Rs. 270

The price of 1 copy $= \frac{\text{Rs. } 270}{3} = \text{Rs. } 90$

The price of 3 copies $= \text{Rs.}90 \times 5$

$= \text{Rs. } 450$

### Next method:

This problem can be solved by using ratios too.

| Numbers of copies | Price (Rs.) |
|---|---|
| 3 | 270 |
| 5 | $x$ (Let) |

Since there is a direct variation between the number of copies and their price, it can be written in proportion as follows.

So, $\frac{3}{5} = \frac{270}{x}$

or, $3x = 270 \times 5$

or, $x = \frac{270 \times 5}{3}$

$\therefore \ x = 450$

Since both variables decrease and increase in direct variation, this ratio can be written as

$$\frac{5}{3} = \frac{x}{270}$$

Therefore, the price of 5 copies is Rs. 450.

## 8.2 Indirect Variation

**Activity 2**

Discuss the question by observing the table given below:

| | | | |
|---|---|---|---|
| Number of workers | 1 | 2 | 5 |
| Time to complete the work | 10 | 5 | 2 |

What is the relationship between the number of workers and the number of days to complete the work?

In the above table, the number of working days seems to decrease as the number of workers increases. This is considered to be indirect variation.

When one of the two quantities decreases, the other quantity increases in the same proportion, and when one quantity decreases, the other quantity increase in the same proportion, then the relation between those quantities is said to be indirect variation.

**Example 2**

**If 12 men can dig a field-in 20 days, how many days does it take for 8 men to dig the same field?**

**Solution**

There are 8 men out of 12 men. (The number of men has been decreased. So it takes a long time to complete the work.)

12 men can dig a field in 20 days.

1 man can dig a field in 20 × 12 days.

8 men can dig a field in $\frac{20 \times 12}{8} = 30$ days.

$\therefore$ 8 men can dig a field in 16 days.

**Next method:**

Here

| Numbers of men ↓ | Working days ↑ |
|---|---|
| 12 | 20 |
| 8 | $x$ (Let) |

There is indirect variation between the working day and the number of men. Therefore, if there are few men, many working days are needed and if there are many men, few working days are needed.

So, $\frac{20}{x} = \frac{8}{12}$

or, $8x = 20 \times 12$

or, $x = \frac{20 \times 12}{8}$

or, $x = 30$

Being indirect variation, it can also be written $\frac{x}{20} = \frac{12}{8}$

$\therefore$ 8 men can dig a field in 16 days.

## Exercise 8

**1. Tick (√ ) if the following statements are correct and cross ( ×) if they are incorrect.**

(a) If the number of items increases, so price of the items also increases. It is a direct variation.

(b) There is a direct variation between the number of people and time taken for the work.

(c) There is a direct variation between taken time for a bus to travel a certain distance and the distance traveled by the bus.

(d) There is indirect variation between the water filling capacity of the drinking water pipe and the time taken to fill.

(e) There is a direct variation between the number of hands and fingers.

2. If the costs of 5 kg sugar is Rs. 450, then what is the cost of 3 kg sugar?

3. A motorcycle can travel 320 kilometers with 8 liters of petrol. How many liters of petrol is required to travel 50 kilometers?

4. If 15 people take 16 days to dig a field, how many days will it take for 8 people to dig the same field?

5. 30 men can plant the crops in 17 days. If Amar wants to complete the same work in 10 days, how many workers will be needed?

6. If a machine of the industry can fill 6600 bottles of beverages in 3 hours, how many bottles can it fill in 8 hours?

7. Rama buys 3 kg lentil and 2 kg sugar for Rs 540. If the cost of 1 kg sugar is Rs. 90, what is the cost of 1 kg lentil?
8. If a cyclist can travel the distance of 15 kilometers per hour, how many meters per minute would his speed be?
9. In a camp, 50 people have enough food for 54 days. How many days will the food last for 60 people?
10. The weight of rice and its cost is shown in the table below. Fill in the blanks based on its weight and cost. (Also show the Procedure.)

| S. N. | Weight of rice (kg) | Cost of rice (Rs.) |
|---|---|---|
| (a) | 10 | 1250 |
| (b) | 1 | |
| (c) | | 375 |
| (d) | 9 | |
| (e) | | 3125 |

## Project Work

Write down the five conditions of direct and indirect variation that are used in our daily life by searching with your senior or from the internet. Present them in the classroom.

## Answer

1. Show the answer to your teacher.

| | | | |
|---|---|---|---|
| 2. Rs. 270 | 3. 1.25 *l* | 4. 30 days | 5. 51 |
| 6. 17600 | 7. Rs. 120 | 8. 250 minutes | 9. 45 days |
| 10. (a) Rs.125 | (b) 3 | (c) 1125 | (d) 25 |

## Miscellaneous Exercise

**1. In a programme hall, 400 chairs are arranged in rows and columns in the square form.**

(a) How many chairs are there in each row?

(b) If 2 / 2 chairs are added in each row and column, how many chairs are required to re-arrange in the square form?

(c) What percentage of seats were added to each row and column?

(d) What percentage of chairs needs to be added to arrange in the square form?

2. The road traffic lights at three different places are changing every 48 seconds, 72 seconds and 108 seconds respectively. If they all change together at 9 o'clock in the morning, find out when they will change the next time again?

**3. Bimala bought 150 eggs in five crates and brought them in cartons. When she came home, she opened the carton of eggs and found 30 eggs were broken.**

(a) If she sells the remaining eggs each at Rs. 15, she will loss 10 %, how much did buy the 150 eggs?

(b) If 5 % profit to get, at what rate should the remaining eggs be sold?

(c) If only 25 eggs were broken, how much would Bimala have to sell each egg to get 5 % profit?

**4. A school building is being painted. 6 workers can paint 520.2 meters of wall in one hour.**

(a) How many meters of wall can 7 workers paint in one hour?

(b) How many hours does it take for 7 workers to paint the 3641.4 meter wall?

5. ***a*, *b*, and c are three integers. If *a* = –25, *b* = 8 and *c* = –4 then prove that:**

(a) $a + (b + c) = (a + b) + c$

(b) $a \times (b + c) = ab + ac$

6. **Find any two integers:**

(a) Whose sum is –5.

(b) Whose difference is –7.

(c) Whose difference is 0.

7. Angel scored 150 marks in the first terminal examination out of 200 full marks. Roshani scored 180 marks out of 300 full marks in this exam. Compare who gets better results.

8. **In the entrance exam, there were a rule to provide (+5) for each correct answer, (–2) for each wrong answer and (0) for failing to answer.**

(a) Ram solved 7 questions in total, out of which 4 are with correct answers and 3 are with wrong answers. How many marks did he get in total?

(b) Ruchita solved the 8 questions in total, out of which 4 are correct and 4 are wrong. How many marks did she get in total?

(c) Who gets more marks? And how many does he/she get more? Find out.

9. **In a quiz contest of the school rules were made to provide (+5) for each correct answer and (–2) for each wrong answer.**

(a) The Red House got 30 marks in which 10 questions were answered incorrectly. Find out how many questions have been answered correctly.

(b) The Green House got (–12) marks out of which 16 questions were answered incorrectly. Find out how many questions have been answered correctly.

(c) Which group answered the most questions correctly? How many more questions did the group answer than the group who answered fewer? Write.

10. On the first day, Anuja read $\frac{1}{5}$ part of a book. She read 40 pages on the second day. If she read $\frac{7}{10}$ parts of the book in two days, find out how many pages does the book have in total.

11. Write any two rational numbers which lie between $\frac{1}{4}$ and $\frac{2}{5}$.

## Answer

1. (a) 20 (b) 84 (c) 10% (d) 21%
2. 9:7:12
3. (a) Rs. 2000 (b) Rs. 17.5
4. (a) 606.9 m (b) 6 hour
5. Show answer to your teacher
6. Show answer to your teacher
7. Anjal
8. (a) 14 (b) 12 (c) Ram, more 2 marks
9. (a) 10 (b) 4 (c) Red house
10. 80
11. $\frac{9}{40}, \frac{19}{80}$

# Lesson 9 Perimeter, Area and Volume

## 9.0 Review



The solid objects given above are of different sizes. Discuss the following questions in appropriate groups about their size, length of the sides, area and length of the surrounding edges.

(a) Write the name of each shape.

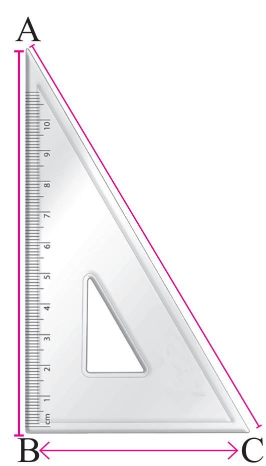
(b) Find the perimeter of the upper surface of each solid object.

(c) How can the surface area of each solid object be calculated?

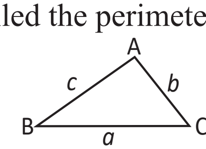
## 9.1 Direct Variation

### Activity 1

Sit in groups with appropriate numbers. Measure the length of the three edges of a set square in your geometry box. Find out the perimeter of the set square.




The sum of the edges around the triangle is called the perimeter of the triangle.

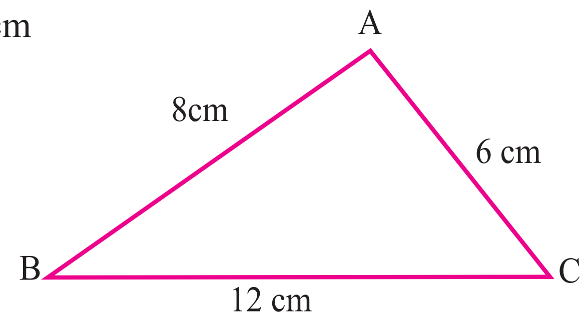



Here, perimeter of $\Delta ABC$,
$P = BC + CA + AB$
or, $p = a + b + c$

## Example 1

**Find he perimeter of the given triangle:**

**Solution**




Here, side of triangle (AB) = 8 cm

Side of triangle (BC) = 12 cm

Side of triangle (AC) = 6 cm

Perimeter of the triangle (P) =?

Now, P= AB + BC+ CA

= 8 cm + 12 cm + 6 cm

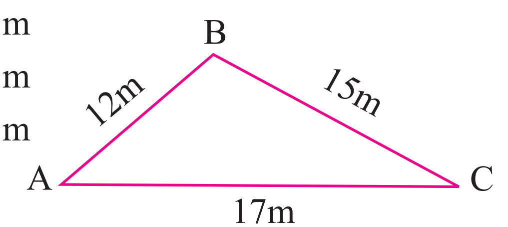
= 26 cm

Therefore, perimeter of the triangle (P) = 26 cm

## Example 2

**If Anupama bought triangular land having its measurement as 15 m, 17 m and 12 m, find out how long wall should she build around it.**

**Solution**




Here, first edge of the land (a) = 15 m

Second edge of the land (b) = 17 m

Third edge of the land (c) = 12 m

Perimeter of the land (P) = ?

Now, P $= a + b + c$

= (15 + 17 + 12) m

= 44 m

The length of the wall around the land is 44 meters.

## Example 3

**Find out the length of the second edge of the triangular park whose total length around it is 400 meters and the length of the first edge is 120 m and the length of the third edge is 180 m.**

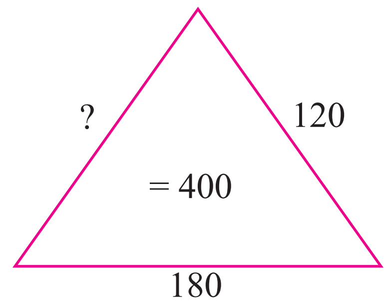
### Solution

Here, perimeter of the park (p) = 400 m

First edge of the park (a) = 120

Third edge of the park (c) = 180 m

Second edge of the park (b) =?




Now, P = $a + b + c$

or, 400 =120 + $b$ +180

or, 400 – 300 = $b$

or, $b$ =100 m

Therefore, the length of the second edge of the park is 100 m

## Example 4

**Find out how much Diya spent if she used wire costing Rs. 20 per meter for fencing 5 times in triangular vegetable field which has the length edges of 10 m, 12 m and 14 m.**

### Solution

The first edge of the vegetable field (a) = 10 m

The second edge of the vegetable field (b) = 12 m

The third edge of the vegetable field (c) = 14 m

Now, P = a + b + c

= (10 + 12 + 14) m = 36 m

The length of the wire used to enclose the fence at a time = 36 m

∴ Length of wire used for fencing at 5 times = 36 × 5 m = 180 m

Cost of the wire per meter = Rs. 20

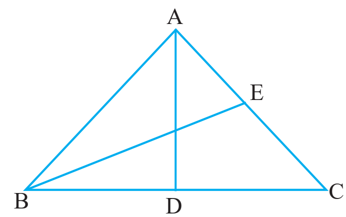
Cost of the wire of 180 m = 180 × 20 m = Rs. 3,600

Therefore, she spent Rs. 3,600 for fencing with wire.

## Example 5

**The shape as shown in the given picture is made by using small sticks. If AD + BE = 7 cm, and AB= BC = CA =7.5 cm, then find the total length of the sticks.**

**Solution**




Here,

AD = BE = 7cm and AB = BC = CA = 7.5cm

Total length of the sticks

= AB + BC + CA + AD + BE

= 7.5 + 7.5 + 7.5 + 7 + 7

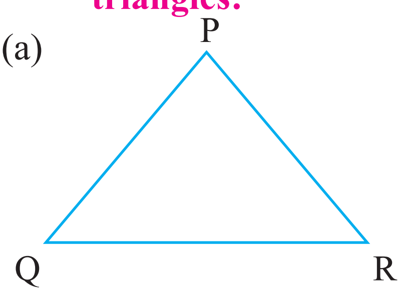
= 36.5 cm

∴ Total length of these sticks is 36.5 cm.

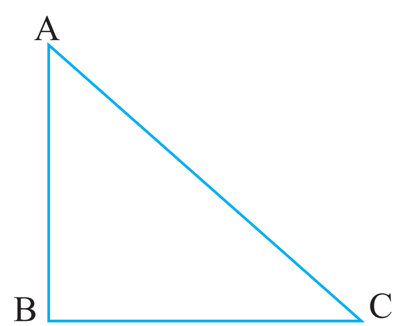
## Exercise 9.1

1. **Find the perimeter by measuring the sides of the following triangles:**

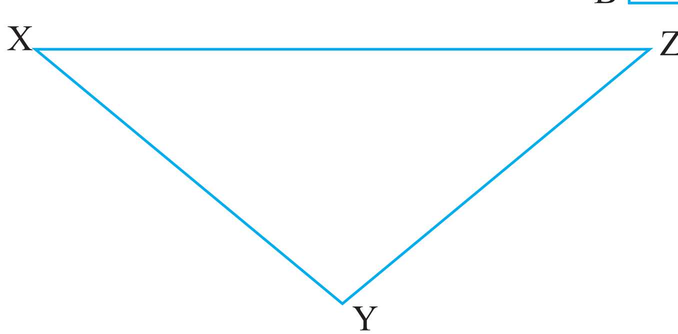
(a)




(b)

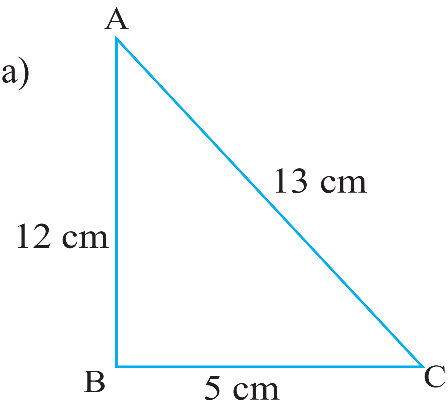



(c)

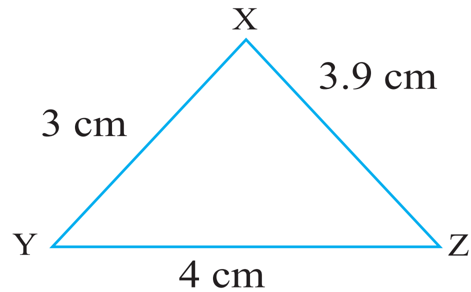


**2. Find the perimeter of the following triangles:**

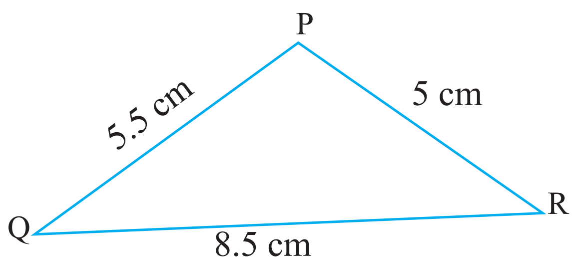
(a)



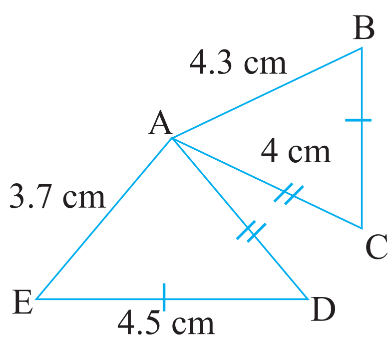

(b)




(c)



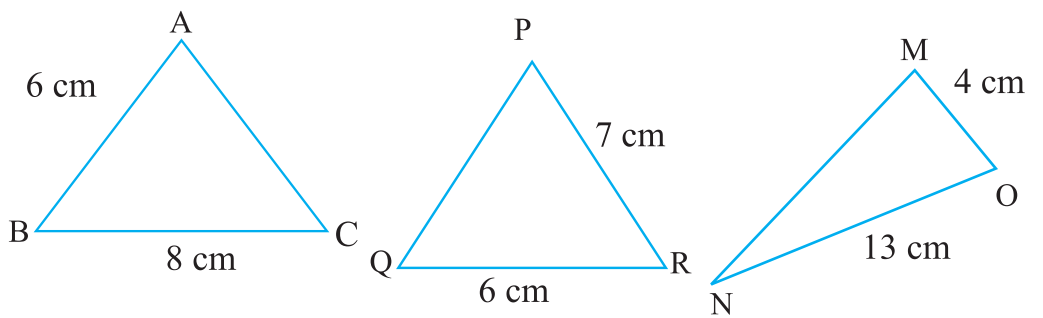

**3. Find the perimeter of the following triangles:**




4. When Ram makes a paper triangle with the length of 40 cm around it, find the length of the third side. If the two sides are 14 cm and 16 cm respectively.
5. Find out how many times it can be fenced with a 540 m long wire to an equilateral triangular land with 18 meters side.

6. Among the three edges of a triangular garden, the length of the first edge is 12 ft and the length of the other two edges of equal size is 7 ft. How much does it cost to enclose the triangular garden 20 times if a plastic rope costs 60 paisa per foot?

7. Find the unknown length of the following triangles.




Perimeter (P) = 19 cm    Perimeter (P) = 23 cm    Perimeter (P) = 28 cm

## Project Work

Set the three wooden sticks upright at some distance on the school's ground and make a triangular shape using the rope. Find the perimeter of the shape and present it in the classroom.

## Answer

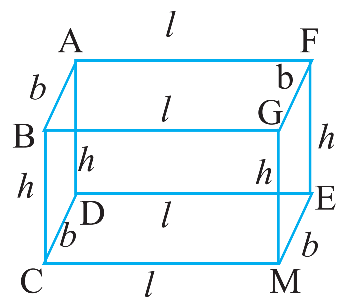
1. Show answers to your teacher
2. (a) 30 cm (b) 10.9 cm (c) 19 cm
3. 12.2 cm, 12.8 cm
4. 10 cm  5. 10 times  6. Rs. 312
7. (a) 5 cm (b) 10 cm (c) 11 cm

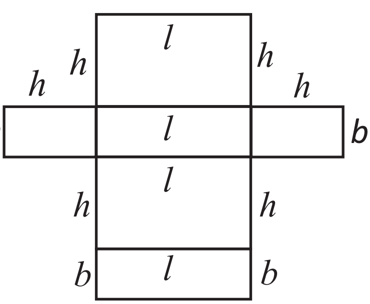
## 9.2 Cuboid and Cube

### 9.2.1 Total Surface Area of Cuboid

**Activity 1**

Take a cuboid and separate its length, breadth and height. Discuss the rectangular surfaces formed in the cuboid. The rectangular surfaces in the cuboid are shown ABCD, ABGF, ADEF, BCMG, CDEM, GFEM respectively.




Here

Area of rectangle ABCD ($A_1$) $= CD \times AD$
$= b \times h = bh$

Area of rectangle ABGF ($A_2$) $= AB \times AF$
$= b \times l = lb$

Area of rectangle ADEF ($A_3$) $= AD \times AF$
$= h \times l = hl$

Area of rectangle BCMG ($A_4$) $= BG \times BC$
$= l \times h = lh$

Area of rectangle CDEM ($A_5$) $= CD \times DE$
$= b \times l = bl$

Area of rectangle GFEM ($A_6$) $= GF \times GM$
$= b \times h = bh$

Hence, total surface area of cuboid (A) $= A_1 + A_2 + A_3 + A_4 + A_5 + A_6$
$= bh + lb + lh + lh + lb + bh$
$= 2lb + 2bh + 2lh$
$= 2(lb + bh + lh)$

The total surface area of cuboid is $= 2(lb + bh + lh)$

## Example 1

Find the total surface area of a cuboid having 4 cm length, 3 cm breadth and 2 cm height.

Here, length ($l$) = 4 cm

breadth (b) = 3 cm

height (h) = 2 cm

Total surface area (A) = ?

Now, total surface area of a cuboid (A) = $2(lb + bh + lh)$

$= 2(4 \times 3 + 3 \times 2 + 4 \times 2)$

$= 2(12 + 6 + 8)$

$= 2 \times 26$

$= 52 \text{ cm}^2$

Therefore. total surface area of cuboid is $52 \text{ cm}^2$.

## Example 2

**If a geometry box has length (l) = 15 cm, breadth (b) = 7 cm and height (h) = 3 cm then find the total surface area of the box.**

### Solution

Here, length ($l$) = 15 cm

breadth (b) = 7 cm

height (h) = 3 cm

Total surface area of the geometry box (A) = ?

Now, according to formula,

Total surface area of the geometry box (A) = $2(lb + bh + lh)$

$= 2(15 \times 7 + 7 \times 3 + 15 \times 3)$

$= 2(105 + 21 + 45)$

$= 345 \text{ cm}^2$

$\therefore$ Total surface area of the geometry box is $345 \text{ cm}^2$.

## Example 3

**Find the breadth of a box having its length of 42 cm, height of 28 cm and total surface area of 7812 cm$^2$.**

**Solution**

Here, length ($l$) = 42 cm

breadth (b) = ?

height (h) = 28 cm

Total surface area of the box (A) = 7812 cm$^2$

Now, according to formula,

Total surface area of the box (A) = $2(lb + bh + lh)$

or, $7812 = 2(42b + 28b + 42 \times 28)$

or, $7812 = 2(70b + 1176)$

or, $7812 = 140b + 2352$

or, $7812 - 2352 = 140b$

or, $\frac{5460}{140} = b$

or, b = 39 cm

Therefore, breadth of the box is 39 cm.

## 9.2.2 Total Surface Area of a Cube

### Activity 2

Take a dice and measure the length of the edge of the dice. You will get the length of all its equal sides.

Here is length (l) = breadth (b) = height (h).

A cuboid having equal length, breadth and height is called a cube.

Now, let $l = b = h = a$

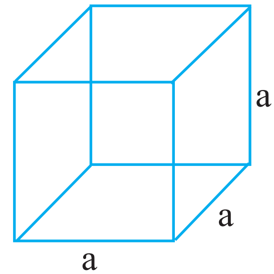
Total surface area of cuboid (A) = $2(lb + bh + lh)$

$= 2(a \cdot a + a \cdot a + a \cdot a)$

$= 2(a^2 + a^2 + a^2)$

$= 6a^2$

Therefore, total surface area of cube is (A) = $6a^2$ square unit.




**Next method**

Area of a surface of the cube = $a \times a = a^2$

Area of 6 surface of the cube = $6a^2$

$\therefore$ Total surface area of cube (A) = $6a^2$

**Example 1**

**The length of one edge of a solid paper toy is 3.5 cm. Find the surface area of the toy.**

**Solution**

Here, length of a side of cube (a) = 3.5 cm

Total surface area of the box (A) = ?

Now, according to formula,

Total surface area of the cube (A) = $6a^2$

$= 6 \times (3.5)^2$

$= 6 \times 12.25$

$= 73.50 \text{ cm}^2$

$\therefore$ Total surface area of the toy is (A) = $73.50 \text{ cm}^2$

## Example 2

**If the surface area of a cubical solid is 54 m², find the length of one edge of the solid.**

**Solution**

Total surface area of the cubical solid (A) = 54 $m^2$

Length of a edge (a) = ?

Now, according to formula, total surface area of the box (A) = $6a^2$

or, $54 = 6a^2$

or, $\frac{54}{6} = a^2$

or, $9 = a^2$

or, $a = \sqrt{9}$

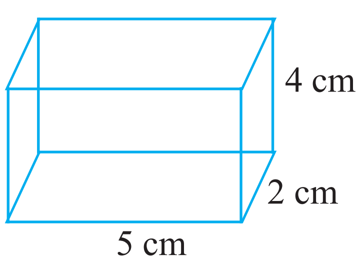
or, $a = 3$ m

∴ The length of one edge of the solid is (a) = 3 m

## Exercise 9.2

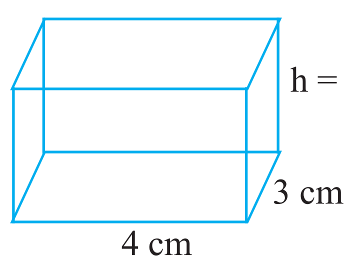
**1. Find the unknown edge or surface area of each solid object given below:**

(a)



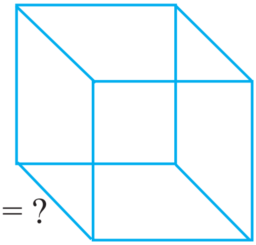

Total surface area (A) = ?

(b)



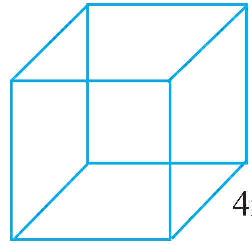

Total surface area (A) = 45 $cm^2$

(c)




Total surface area (A) = 216 $cm^2$

(d)




Total surface area (A) = ?

2. (a) If the cuboid has length ($l$) = 10 cm, breadth (b) = 8 cm and total surface area (A) =376 $cm^2$, find the height (h) of the cuboid.

   (b) b) The total surface area of one cartoon biscuit is 9400 $cm^2$. If its length is 50 cm and breadth is 30 cm , find its height.

3. (a) If the total surface area of a solid is 726 $cm^2$, find the length of one edge of the solid. What is the area covered by the solid on the ground, find out.

   (b) If the total surface area of a solid is 864 $cm^2$, find the length of the side of the solid. When removing its lid, find out how much area can be painted on the outside of it.

**4. A duster having 7 cm length and 8 cm breadth is placed on the table and covered 80 $cm^2$ of the surface of the table.**

   a) Find the length of the duster.

   b) Find the total surface area of the duster.

**5. Measure the length of the edges of the surface of cuboid and cubical solids as given below and find the total surface area of the solid**

(a)

(b)

## Project Work

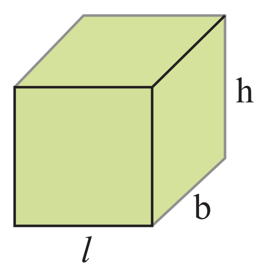
Make a cuboid from the chart paper. Find the total surface area by measuring the edge of the cuboid and present it in the classroom.

## Answer

1. (a) 76 $cm^2$ (b) 1.5 cm (c) 96 $cm^2$ (d) 6 cm
2. (a) 6 cm (b) 40 cm
3. (a) 11 cm, 121 $cm^2$ (b) 12 cm, 720 $cm^2$
4. (a) 10 cm, 412 $cm^2$
5. Show answers to your teacher.

## 9.3 Volume of Cuboid and Cube

### Activity 1




If the edge of a cuboid shape container have the length (l) =15 cm, breadth (b) = 14 cm and height (h) = 13 cm, how much colors can this colour pot contain? Discuss with your classmates.

Here, length ($l$) = 15 cm,

breadth (b) = 14 cm

height (h) = 13 cm

Now, Volume (V) = $l \times b \times h$

= (15 × 14 × 13)

= 2730 $cm^3$

∴ It contains 2730 $cm^3$.

## Activity 2

The length of an edge of a cubical pot of rice is 60 cm. How much rice does this pot contain? Discuss with classmates.

Here, since all the edges of a cubical pot are equal, each side is 60 cm in length, breadth and height.

Now, capacity of this pot = length × breadth × height

$= 60 \times 60 \times 60$

$= 216000 \text{ cm}^3$

Therefore, the volume of this pot (V) = $216000 \text{ cm}^3$

Hence, the volume of cubical solid object is $(V) = l \times l \times l$

$V = l^3$

## Example 1

**The length, breadth and height of a cuboid shape sugar pot is 3 m, 2 m and 1m respectively. Find out how much sugar this sugar pot contains.**

**Solution**

Here, length $(l)$ = 3 m,

breadth (b) = 2 m

height (h) = 1 m

Now, Volume of the pot (V) $= l \times b \times h$

$= 3 \times 2 \times 1$

$= 6 \text{ m}^3$

∴ It contains $6 \text{ m}^3$ sugar.

## Example 2

**A rectanlar tank contains 600 $l$ of water. Find the breadth if the length of the tank is 200 cm and height is 5 cm.**

**Solution**

Here, volume (V) = 600 $l$

$= \frac{600}{1000} = 0.6 \text{ m}^3$ $\quad \left[\because 1\ l = \frac{1}{1000} \text{ m}^3\right]$

length $(l)$ = 200 cm = 2 m

height (h) = 50 cm = 0.5 m

breadth (b) = ?

Now, according to formula (V) = $l \times b \times h$

or, $0.6 = 2 \times b \times 0.5$

or, $0.6 = b$

or, $b = \frac{6}{10} = 0.6 \text{ m} = 60 \text{ cm}$

or, $b = 60$

Therefore, length of the container is 60 cm.

## Example 3

**The length of cuboid is three times of its breadth. If its height and volume are 8 cm and 864 cm³ respectively, then find its total surface area.**

**Solution**

Here, volume (V) = 864 cm³

height (h) = 8 cm

let, breadth $(b)$ = $x$ cm

$\therefore$ length $(l)$ = $3x$ cm

Now, according to formula, V = $l \times b \times h$

or, $864 = 3x \times x \times 8$

or, $864 = 24x^2$

or, $x^2 = \dfrac{864}{24} = 36$

or, $x = 6$

or, b = 6 cm

or, $l = 3x = 3 \times 6 = 18$ cm

Now, the total surface area = $2(lb + bh + lh)$

$= 2(18 \times 6 + 6 \times 8 + 18 \times 8)$

$= 2(108 + 48 + 144)$

$= 600 \text{ cm}^2$

Therefore, total surface of the cuboid is 600 $\text{cm}^2$.

## Example 4

**If the length of one edge of a cubical water tank is 1.5 m, then findout how many liters of water it can hold.**

**Solution**

Here, the length of one edge of a cubical water tank (a) = 1.5 m

Volume (V) = ?

Now, according to formula (V) = $a^3$

$= (1.5)^3$

$= 3.375 \text{ m}^3$

$= 3.375 \times 1000\ l$ $\quad \left[\because 1 \text{ m}^3 = 1000\ l\right]$

$\therefore$ V = 3375 $l$

Therefore, this pot holds 3375 $l$ water .

## Example 5

**A cubic tank holds 216,000 liters of water. Find the surface area of its surface.**

### Solution

Here, volume of cube (V) = 216000 $l$

$= \frac{216000}{1000} \text{ m}^3 = 216 \text{ m}^3$ $\quad \left[\because 1\, l = \frac{1}{1000} \text{ m}^3\right]$

Length of the side of cube (a) = ?

Now, according to formula (V) = $a^3$

or, $216\text{m}^3 = a^3$

or, $a = \sqrt[3]{216}$

or, $a = 6$ m

Now, total surface area (A) = $6a^2$

$= 6 \times 6^2$

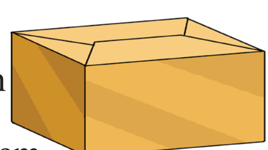
$= 216 \text{ m}^2$

Therefore, the total surface area of the cube is 216 m$^2$.

## Exercise 9.3

**1. Find the volume of the solid objects given below:**

(a)



5 cm

6 cm

16 cm

(b)

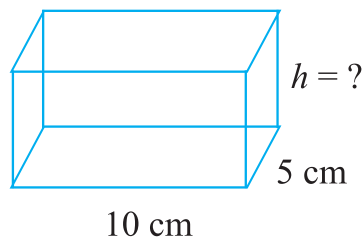

3.5 cm

1.5 cm

5 cm

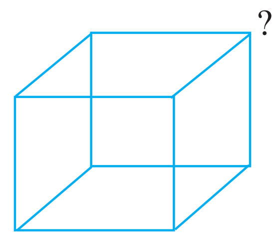
**2. Find the unknown edges in the solid objects given below:**

(a)




V = 450 cm$^3$

(b)




V = 343 cm$^3$

3. A room is 5 m long, 4 m wide and 3 m high. Find the volume of the room.

4. The length of a seminar hall is twice its height. If its length is 8 m and volume 576 m$^3$. Find the total surface area of the room.

5. The length of one edge of a dice is 9 cm. Find the volume of the dice.

6. (a) The volume of a cubical box is 512 cm$^3$. Find the total surface area of the box.

   (b) The volume of a cubical box is 125 cm$^3$. Find the length of an edge and total surface area of the solid.

**7. A rectangular piece of gold measuring 4 cm in length, 2 m in width and 1 cm in height is melted down to a cube then**

   (a) Find the volume of a cubical piece of gold.

   (b) Find the total surface area of a cubical piece of gold.

8. The milk pot is 32 cm long, 16 cm wide and 8 cm high. How many times does it take to make it empty when it is pulled out by a 8 cm long cubical pot? Find out.

9. (a) When constructing a rectangular tank containing 64,000 *l* of water for a village, the length is 8 m and breadth 4 m. Find out, how deep should the tank be made?

   (b) When constructing a cubical tank of water with a capacity of 216,000 *l* for a school, what is the area under the base of the water tank?

## Project Work

Find the volume of cubical solid by collecting from around your home and discuss the results of your work in the classroom.

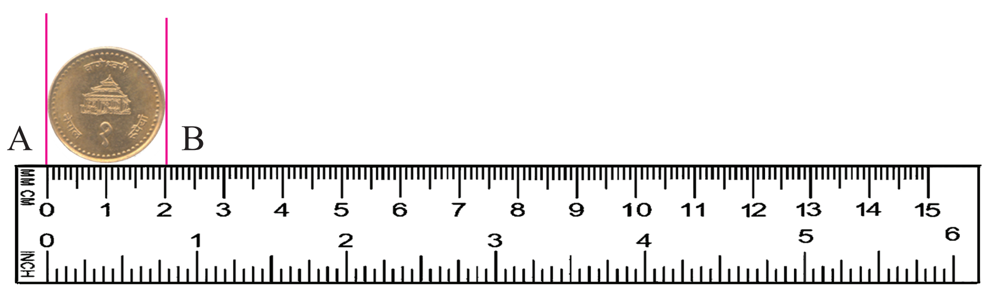
## Answer

| | | | |
|---|---|---|---|
| 1. | (a) 480 $cm^3$ <br> (b) 7 cm | (b) 26.25 $cm^3$ | 2. (a) 9 cm <br> 3. 60 $m^3$ 4. 432 $m^2$ |
| 5. | 729 $cm^3$ | 6. (a) 384 $cm^2$ | (b) 5 cm, 150 $cm^2$ |
| 7. | (a) 8 $cm^3$ | (b) 24 $cm^2$ | 8. 8 times |
| 9. | (a) 2 m | (b) 36 $m^2$ | |

## 9.4 Relation between Circumference and Diameter of Circle and its Uses

### Activity 1

Take a coin and tie the coin around with a thread and measure the length of the thread or turn a coin on scaled surface one round and measure its distance.




Now, find the distance between the point A and B in the picture.

Here, the distance between points A and B = 2 cm

Length of the thread ($l$) = 6.28 cm

Now dividing the length of the thread CD by AB is $\frac{6.28}{2} = 3.14$

Dividing the circumference of both by their diameter becomes approximately 3.14.

or, $c = \pi d = 2\pi r$ [ $\because$ diameter (d) = 2 × radius.]


Similarly, take a bangle. Find its diameter and circumference and its ratio.

Is the value of the circumference of the coin divided by its diameter equal to the value of the circumference of the coin divided by its diameter?

3.14 is a constant value. It is denoted by Greek letter 'π'.

Therefore, $\frac{c}{d} = \pi$

## Example 1

**If the diameter of a circle is 14 cm, then find the radius and circumference of the circle. ($\pi = \frac{22}{7}$)**

**Solution**

Here, diameter of the circle ($d$) = 14 cm

$\therefore$ Radius of the circle ($r$) $= \frac{d}{2}$ $\quad$ [$\because d = 2r$]

$= \frac{14}{2}$

$= 7$

Now, circumference of the circle ($c$) $= 2\pi r$

$= 2 \times \frac{22}{7} \times 7$

$= 44$ cm

Therefore, radius of the circle = 7 cm and length of the circle = 44 cm.

## Example 2

**The circumference of a circular ring of gold is 7 cm. Find the diameter of the ring.** $(\pi = \frac{22}{7})$

**Solution**

Here, circumference of the circle (C) =7 cm

Diameter of the circle (d) = ?

According formula,

$$c = \pi d$$

$$d = \frac{c}{\pi}$$

$$= \frac{7}{\frac{22}{7}}$$

$$= \frac{7 \times 7}{22}$$

$$= 2.23 \text{ cm}$$

Therefore, the diameter (d) = 2.23 cm

## Example 3

**The diameter of a circular fish pond is 56 m. Find out how long wires it takes to wrap it around 5 times.** $(\pi = \frac{22}{7})$

**Solution**

Diameter of the fish pond (d) = 56 m

Circumference of the fish pond (C) =?

According to the formula,

$$c = \pi d$$

$$= \frac{22}{7} \times 56 \text{ m}$$

$$= 176 \text{ m}$$

Therefore, radius of the circle = 7 cm and length of the circle = 44 cm.

The wire needed to encircle the barbed wire once $= 176$ m

$\therefore$ Wire required to enclose barbed wire 5 times $= 176 \times 5$ m

$= 880$ m

Therefore, length of the wire is 880 m.

## Example 4

**When Ram crosses a distance of 440 meters from a bicycle, the wheel turns 100 times. Find the radius of the wheel. ($\pi = \frac{22}{7}$)**

**Solution**

Here, the distance covered by turning the wheel 100 times = 440 m

Radius of the wheel (r) = ?

Circumference of the wheel (C) $= \frac{440}{100} = 4.4 \text{ m} = 4.4 \times 100 \text{ cm} = 440 \text{ cm}$.

Now, according to the formula,

$$C = \pi d$$

or, $440 = \frac{22}{7} \times d$

or, $d = \frac{440 \times 7}{22}$

or, $2r = 140$

or, $r = \frac{140}{2} = 70$ cm

Therefore, radius of the circle = 70 cm

## Exercise 9.4

1. **Find the circumference of the circle based on the measurements given below: ($\pi = \frac{22}{7}$)**

   (a) $r$ = 3.5 cm  (b) $r$ = 49 cm  (c) $r$ = 10.5 cm

   (d) $d$ = 70 m  (e) $d$ = 17.5 cm  (f) $d$ = 56 m

2. **Find the radius of the circle from the length of the circumference of the circle given below: ($\pi = \frac{22}{7}$)**

   (a) $c$ = 176 cm  (b) $c$ = 308 cm  (c) $c$ = 616 cm

   (d) $c$ = 660 m  (e) $c$ = 242 cm  (f) $c$ = 330 m

3. (a) If a goat tied with a 14 meter long rope stretches the rope and walks around, how far does the goat walk in 5 times? Find out.

   (b) How many meters does the wheel of a 77 cm diameter of car travel at 50 times? Find out.

4. (a) How many times does the wheel of a bicycle with a radius of 35 cm have to be rolled to cover a distance of 44 meters? Find out.

   (b) Find out how many times can cover a distance of 17.6 km by running around in a circular pond with a diameter of 140 meters.

5. (a) If the wheel of a bus travels a distance of 44 meters by rolling 20 times, find the radius of the wheel.

   (b) Find out the approximate diameter of the pond if Sita walks a distance of 1 km 980 m while walking 15 times around a circular pond.

## Project Work

Find the length of the circumference by measuring the diameter of the circular objects around your house. Also find the ratio of circumference and diameter and discuss the results in the class.

## Answer

| | | | |
|---|---|---|---|
| 1. (a) 22 cm | (b) 308 cm | (c) 66 cm | (d) 220 m |
| (e) 55 m | (f) 176 m | | |
| 2. (a) 28 cm | (b) 49 m | (c) 98 m | (d) 105 m |
| (e) 38.5 cm | (f) 52.5 m | | |
| 3. (a) 440 m | (b) 121 m | | |
| 4. (a) 20 k6s | (b) 40 k6s | | |
| 5. (a) 35 cm | (b) 42 cm | | |

## Miscellaneous Exercise

1. If Ram bought a triangular piece of land with 30 feet, 24 feet and 27 feet edges, find out the length around the land.

2. If the perimeter of an equilateral triangle is 23.25 cm, find the length of the one side.

3. Ram bought a 440 meter long net for Rs 35,200. He used the net to fence his triangular land where the length of the three sides of the land is 9 m, 8 m and 5 m respectively. How many times has he surrounded the land? Find out how much it costs to fence the once.

4. (a) If a cubical tank holds 343,000 l of water, then calculate the total surface area of the tank.

   (b) If the total surface area of a cubical tank is 1776 $m^2$, find out how many liters of water it can hold.

5. The length of the cuboid is double of its breadth. If its height is 10 cm and volume is 8000 $cm^3$ then find the total surface area.

6. If the length of a room is 8 m and height is 4 m, total surface area is 304 $m^2$, find the volume of the room.

7. The length of a cuboid is twice its breadth. If the height of the cuboid and the area of the total surface area are 10 cm and 736 $cm^2$ respectively. Find the volume of the cuboid.

8. **The length of a room is three times of its breadth. If the height of the room is 2 m and the volume is 96 $m^3$ then,**

   (a) Find the length and breadth of the room.

   (b) Find the total surface area of the room.

9. A 44 meter long wire is bent to form a triangle. If the length of the two edges is 16 m and 17 m, find the length of the remaining edge.

10. If the total surface area of a cube is 600 $cm^2$, find the volume of the cube.

11. The volume of a cube is 1728 $cm^3$. The total surface area of a cube is equal to the total surface area of the cuboid. Find the length, if the cuboid is 8 cm and the breadth is 12 m.

12. The total surface area of a cubical piece of silver is 150 $cm^2$. If it is melted to make a cuboid with 5 cm breadth and 4 cm height, find the length of the cuboid.

13. Find the radius of the wheel, if the wheel of a truck travels a distance of 2.2 km in 400 times.

14. A rectangular tank filled with water has 1 m length, 90 cm breadth and 60 cm height. Find out, how many times the tank becomes completely empty when the water from the tank is pumped out by a cubical vessel with a 30 cm edge.

**15. If a room is 8 meters long, 6 meters wide and 2 meters high,**

(a) Find out how much it costs to paint the total surface of the room at the rate of Rs. 50 per square meter.

(b) If 0.5 $m^2$ space is given per student, find out how many students are occupied in that room.

## Answer

| | | | | | |
|---|---|---|---|---|---|
| 1. | 81 ft | 2. | 7.75 cm | 3. | 20 times |
| 4. | (a) 294 $m^2$ | (b) 2744000 $l$ | | | |
| 5. | (a) 2800 $cm^2$ | (b) 320 $m^3$ | | 6. | 1280 $cm^3$ |
| 7. | $l = 12$ m, b = 4 m, h = 2 m and A = 160 $m^2$ | | | | |
| 8. | 11 m | 9. | 1000 $m^3$ | 10. | 16.8 cm |
| 11. | 6.25 cm | 12. | 87.5 cm | 13. | 20 times |
| 14. | | 15. | (a) Rs. 7600 | (b) | 192 |

Lesson 10

# Indices

## 10.0 Review

Study the following activity and discuss about it.

The pattern of multiplication of same number is given below. Complete it.

| **Continuous multiplication method** | **method of reading** |
|---|---|
| $3 \times 3$ | $3^2$ = 3 to the power 2 |
| $3 \times 3 \times 3$ | $3^3$ = 3 to the power 3 |
| $3 \times 3 \times 3 \times 3$ | $3^4$ = 3 to the power 4 |
| ... | ... |
| $a \times a \times a \times a \times \ldots n$ times | $a^n$ = $a$ to the power $n$ |

In $a^n$, $a$ is base, $n$ is power and read as a power $n$.

In $3^5$ base is 3 and power is 5.

In $2^3$, base is 2 and power is 3.

Base $\longrightarrow$ $\mathbf{2^3}$ $\longleftarrow$ Index

So, the indices is used to multiply a number by it in many times.
In $a^n$, $a$ is base, $n$ is power and read as $a$ power $n$.

## 10.1 Laws of Indices

**Activity 1**

Study the following pattern and discuss about it.

$$a^1 \times a^1 = a^{1+1} = a^2$$
$$a^2 \times a^1 = a^{2+1} = a^3$$
$$a^3 \times a^1 = a^{3+1} = a^4$$
$$\ldots$$
$$a^m \times a^n = a^{m+n}$$

**Rule 1 :** The power is added when the base is same.

$$a^m \times a^n = a^{m+n}$$

## Activity 2

Study the following pattern and discuss about it.

$$2^2 \div 2^1 = \frac{2^2}{2} = \frac{2 \times 2}{2} = 2 = 2^1 = 2^{2-1}$$

$$3^3 \div 3^1 = \frac{3^3}{3} = \frac{3 \times 3 \times 3}{3} = 9 = 3^2 = 3^{3-1}$$

$$5^5 \div 5^2 = \frac{5^5}{5^2} = \frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5} = 125 = 5^3 = 5^{5-2}$$

...

$$a^m \div a^n = a^{m-n}$$

**Rule 2:** When the indices of the same base are divided, the power of denominator is subtracted from numerator.

$$a^m \div a^n = a^{m-n}$$

## Activity 3

Study the following pattern and fill in the blanks.

$$2 \div 2 = \frac{2}{2} = 2^{1-1} = 2^0 = 1$$

$$3^3 \div 3^3 = \frac{3^3}{3^3} = 3^{3-3} = 3^0 = 1$$

$$4^3 \div 4^3 = \frac{4^3}{4^3} = 4^{3-3} = 4^0 = 1$$

$5^3 \div 5^3 =$ ......................................

$6^3 \div 6^3 =$ ......................................

...

$a^m \div a^m =$ ......................................

**Rule 3 :** If the power of any number except zero is zero (0), the value is 1. It means $a^{\circ} = 1$, where $a \neq 0$

## Example 1

### Write the following factors in indices form.

(a) $5 \times 5 \times 5 \times 5 \times 5$

**Solution**

Here $5 \times 5 \times 5 \times 5 \times 5$

$= 5^5$

(b) $(-3y) \times (-3y) \times (-3y) \times (-3y) \times (-3y) \times (-3y)$

**Solution**

Here $(-3y) \times (-3y) \times (-3y) \times (-3y) \times (-3y) \times (-3y)$

$= (-3y)^6$

## Example 2

### Find the multiple of:

(a) $2^3 \times 2^{-2}$ (b) $(3a)^4 \times (3a)^3 \times (3a)^{-7}$

**Solution**

Here,

(a) $2^3 \times 2^{-2}$

$= 2^{3-2}$

$= 2^1$

$= 2$

(b) $(3a)^4 \times (3a)^3 \times (3a)^{-7}$

$= (3a)^{4+3-7}$

$= (3a)^0$

$= 1$

## Example 3

**Simplify:**

(a) $(a+b)^3 \times (a+b)^5$ (b) $(3xy)^5 \div 9x^2y^2$ (c) $\dfrac{10a^4 \times 15a^5}{75a^8}$

**Solution**

Here,

(a) $(a+b)^3 \times (a+b)^5$

$= (a+b)^{3+5}$

$= (a+b)^8$

(b) $(3xy)^5 \div 9x^2y^2$

$= \dfrac{3^5x^5y^5}{9x^2y^2}$

$= \dfrac{3^5}{9} \times \dfrac{x^5}{x^2} \times \dfrac{y^5}{y^2}$

$= 3^{5-2}\,x^{5-2}\,y^{5-2}$

$= 3^3\,x^3\,y^3$

$= 27\,x^3\,y^3$

(c) $\dfrac{10a^4 \times 15a^5}{75a^8}$

$= \dfrac{2 \times 5 \times a^4 \times 3 \times 5 \times a^5}{3 \times 5 \times 5 \times a^8}$

$= \dfrac{2 \times 3 \times 5^2 \times a^4 \times a^5}{3 \times 5^2 \times a^8}$

$= 2 \times 3^{1-1} \times 5^{2-2} \times a^{4+5-8}$

$= 2 \times 3^0 \times 5^0 \times a^{9-8}$

$= 2 \times 1 \times 1 \times a^1$

$= 2a$

## Example 4

**If a+b+c =0, find the value of $x^{a-b} \times x^{a+b} \times x^{b+c} \times x^{b-c} \times x^{c+a} \times x^{c-a}$**

**Solution**

$x^{a-b} \times x^{a+b} \times x^{b+c} \times x^{b-c} \times x^{c+a} \times x^{c-a}$

$= x^{a-b+a+b} \times x^{b+c+b-c} \times x^{c+a+c-a}$

$= x^{2a} \times x^{2b} \times x^{2c}$

$= x^{2a+2b+2c}$

$= x^{2(a+b+c)}$

$= x^{2\times 0}$

$= x^{0}$

$= 1$

## Exercise 10

**1. Express the following continued multiplication into power (indices).**

(a) $3 \times 3 \times 3 \times 3 \times 3$

(b) $4 \times 4 \times 4 \times 4 \times 4 \times 4$

(c) $x \times x \times x \times x$

(d) $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

(e) $(-2a) \times (-2a) \times (-2a) \times (-2a) \times (-2a)$

(f) $\left(-\frac{1}{a}\right) \times \left(-\frac{1}{a}\right) \times \left(-\frac{1}{a}\right) \times \left(-\frac{1}{a}\right)$

**2. Express the following indices into continued multiplication form.**

(a) $3^3$ (b) $(-2)^4$ (c) $(3x)^5$ (d) $\left(\frac{1}{2}\right)^6$

**3. Find the factors of the following and write in indices form.**

(a) 128 (b) 243 (c) 625

(d) 343 (e) $\frac{1}{10000}$ (f) $\frac{1}{1728}$

**4. Find the value of:**

(a) $3 \times 2^2$ (b) $5^2 \times 3^3$ (c) $7^2 \times 2^3$

(d) $(-5)^3 \times (-2)^4$ (e) $(a^4) \times (a^{-4})$ (f) $\frac{2^4}{2^3}$

**5. Simplify:**

(a) $p^{b-c} \times p^{a-b} \times p^{c-a}$

(b) $m^{a-b} \times m^{a+b} \times m^{b+c} \times m^{b-c} \times m^{c+a} \times m^{c-a}$

**6. If $x + y + z = 0$ prove that**

$p^{x-y} \times p^{x+y} \times p^{y+z} \times p^{y-z} \times p^{z+x} \times p^{z-x} = 1$

**7. If $x = 1$, $y = 2$ / $z = -1$ find the value of the following.**

(a) $x^3$ (b) $y^x$ u_ $z^{(xy)}$ (d) $(xy)^{-2}$

(e) $3^x \times 2^y$ (f) $(xyz)^{-1}$ (g) $3y^x z^2$

## Answer

1. (a) $3^5$ (b) $4^6$ (c) $x^4$

(d) $\left(\frac{1}{3}\right)^7$ (e) $(-2a)^5$ (f) $\left(-\frac{1}{a}\right)^4$

2. (a) $3 \times 3 \times 3$ (b) $(-2) \times (-2) \times (-2) \times (-2)$

(c) $3x \times 3x \times 3x \times 3x \times 3x$

(d) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

3. (a) $2^7$ (b) $3^5$ (c) $5^4$

(d) $7^3$ (e) $\frac{1}{2^4 \times 5^4}$ (f) $\frac{1}{3^3 \times 2^6}$

4. (a) 12 (b) 675 (c) 392

(d) –2000 (e) 1 (f) 2

5. (a) 1 (b) $m^{2a+2b+2c}$ 6. Show the answer to your teacher.

7. (a) 1 (b) 2 (c) 1 (d) $\frac{1}{4}$

(e) 12 (f) $\frac{1}{2}$ (g) 6

# Lesson 11 Algebraic Expression

## 11.0 Review

Discuss on the following questions.

(a) How many terms are there in $(3x^2 + 3x)$ ?

(b) Are the terms in $(3x + 7x)$ like or unlike?

(c) What does 9 represent in $(3x + 9)$?

(d) What does 3, $x$ and 2 represent in $3x^2$?

1. If the base and power of the terms are same it is called like terms and if the base and power of the terms are different it is called unlike terms.
2. Like terms can be added or subtracted to each other. When like terms are multiplied to each other, the power of variable is added. Constant is kept in front of variable.
3. The base remains same and power is subtracted when the term are in division form.

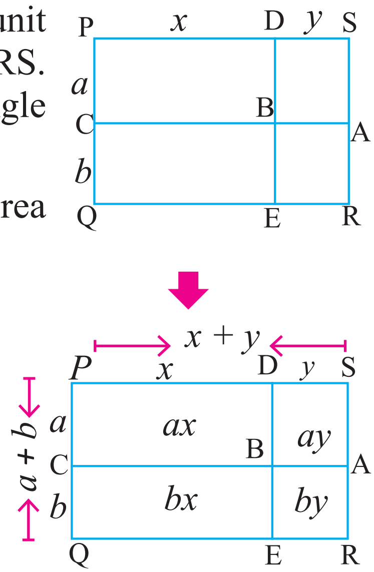
The following are the basic terms which are mainly used in algebraic expression.

| | | |
|---|---|---|
| **Constant** | : | The quantity which are always fixed or same is called constant. |
| **Variable** | : | If the value of quantity is different by its value, it is called variable. |
| **Terms** | : | When the constant or variable or both are connected by multiplication or division sign, it is called terms. |
| **Coefficient** | : | The constant number which multiplies is the variable is called coefficient. |

## 11.1 Multiplication of Binomial Expression by Binomial Algebraic Expression

### Activity 1

In the given figure, the length is $(x + y)$ unit and breadth is $(a + b)$ unit of a rectangle PQRS. Discuss in group and find the area of rectangle PQRS.




Here, area of rect. PQRS = Area of PCBD + Area of DBAS + Area of CQEB + Area of BERA.

Area of rect. PQRS = $(ax+ay+bx+by)$ sq. unit.

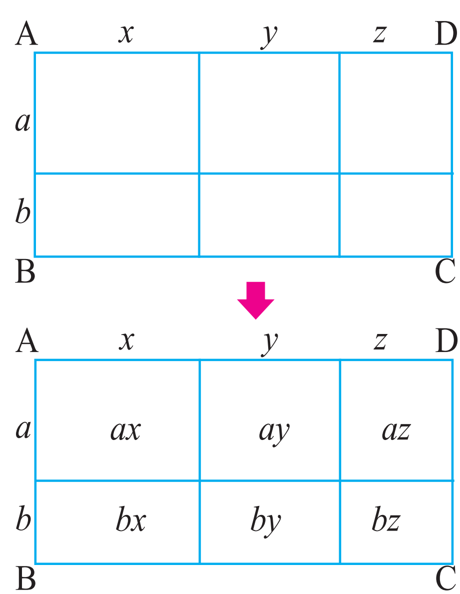
Hence, $(a + b)(x + y) = (ax + ay + bx + by)$

When the two binomial expressions are multiplied to each other, each terms of second expression are multiplied by each term of first expression.

## 11.2 Multiplication of Trinomial Expression by Binomial Algebraic Expression

### Activity 2

In the given figure, the length is $(x + y + z)$ unit and breadth is $(a + b)$ unit of a rectangle ABCD. How can we find the area of rectangle ABCD? Discuss in class.




Here Area of rectangle ABCD = $l \times$ b

$= (x + y + z) \times (a + b)$ sq.unit

Now area of rectangle ABCD = Area of 6 small rectangle

$= (ax + ay + az + bx + by + bz)$ sq. unit

Therefore, $(a + b)(x + y + z) = (ax + ay + az + bx + by + bz)$

When the trinomial expression are multiplied by binomial expression to each other, each terms of second expression are multiplied by each term of the first expression.

### Example 1

**Simplify :**

$$3x(x + y) - 2y(x - y) + 4(xy - x^2)$$

**Solution**

Here $3x(x + y) - 2y(x - y) + 4(xy - x^2)$

$= 3x^2 + 3xy - 2xy + 2y^2 + 4xy - 4x^2$

$= -x^2 + 5xy + 2y^2$

### Example 2

**Find the area of rectangular plot having length (x + 2y) m and breadth $(3x - y)$ m.**

**Solution**

Here, Length of land $(l) = (x + 2y)$ m

Breadth of land (b) $= (3x - y)$ m

Area of land (A) $= l \times b$

$= (x + 2y) \times (3x - y)$

$= x(3x - y) + 2y(3x - y)$

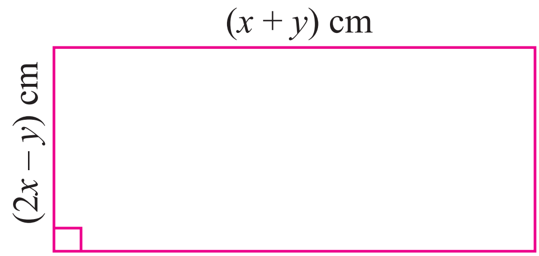
$= 3x^2 - xy + 6xy - 2y^2$

$= (3x^2 + 5xy - 2y^2)\ m^2$

Hence, area of land is $(3x^2 + 5xy - 2y^2)\ m^2$.

## Example 3

**Find the area of the given figure.**




**Solution**

Length ($l$) = $(x + y)$ cm

Breadth (b) = $(2x - y)$cm

Area (A) = $l \times b$

$= (x + y) \times (2x - y)$

$= x(2x - y) + y(2x - y)$

$= 2x^2 - xy + 2xy - y^2$

$= (x^2 + xy - y^2)$ cm$^2$

## Example 4

**Multiply:**

(a) $(5x - 2y) \times (7x - 2y)$

(b) $(2x - y) \times (x + 2y - 3z)$

**Solution**

Here,

(a) $(5x - 2y) \times (7x - 2y)$

$= 5x(7x - 2y) - 2y(7x - 2y)$

$= 35x^2 - 10xy - 14xy + 4y^2$

$= 35x^2 - 24xy + 4y^2$

(b) $(2x - y) \times (x + 2y - 3z)$

$= 2x(x + 2y - 3z) - y(x + 2y - 3z)$

$= 2x^2 + 4xy - 6xz - xy - 2y^2 + 3yz$

$= 2x^2 + 3xy - 6xz - 2y^2 + 3yz$

## Example 5

**If the length and breadth of a rectangular garden are $(2a - b + c)$m and $(a + 2b)$m, find its area.**

**Solution**

Here, length of garden $(l) = (2a - b + c)$ m

breadth of garden $(b) = (a + 2b)$ m

Now area of garden (A) $= l \times b$

$= (2a - b + c) \times (a + 2b)$

$= a(2a - b + c) + 2b(2a - b + c)$

$= 2a^2 - ab + ac + 4ab - 2b^2 + 2bc$

$= (2a^2 + 3ab + 2bc + ac - 2b^2)$ m$^2$

Therefore, area of rectangle $(2a^2 + 3ab + 2bc + ac - 2b^2)\ m^2$

## Example 6

**Find the product of $(5x - 3)$ and $(3x + 4)$. If $x = 2$, find the value of the product.**

**Solution**

Here, the product of $(5x - 3)$ and $(3x + 4) = (5x - 3)(3x + 4)$

$= 5x(3x + 4) - 3(3x + 4)$

$= 15x^2 + 20x - 9x - 12$

$= 15x^2 + 11x - 12$

Put $x = 2$

$15x^2 + 11x - 12$

$= 15(2)^2 + 11 \times 2 - 12$

$= 60 + 22 - 12$

$= 82 - 12$

$= 70$

## Example 7

**If the length and breadth of a rectangular plot is $(3a + 2b)$m and $(2a - b + 3c)$m :**

(a) Find the area of plot.

(b) If $a = 2$, $b = 2$, $c = 1$, find the actual area of land.

**Solution**

Here,

(a) length of plot $(l) = (3a + 2b)$m

breadth of plot $(b) = (2a - b + 3c)$m

Area of land (A) = ?

Area of land (A) $= l \times b$

$= (3a + 2b) \times (2a - b + 3c)$

$= 3a\,(2a - b + 3c) + 2b\,(2a - b + 3c)$

$= 6a^2 - 3ab + 9ac + 4ab - 2b^2 + 6bc$

$= (6a^2 + ab + 9ac + 6bc - 2b^2)\ \text{m}^2$

(b) If $a = 2$, $b = 2$ and $c = 1$

Area of land $= (6a^2 + ab + 9ac + 6bc - 2b^2)$

$= 6 \times 2^2 + 2 \times 2 + 9 \times 2 \times 1 + 6 \times 2 \times 1 - 2 \times 2^2$

$= 24 + 4 + 18 + 12 - 8$

$= 58 - 8$

$= 50\ \text{m}^2$

Hence, area of land $= 50\ \text{m}^2$

## Exercise 11.1

**1. Simplify :**

(a) $3x(x + 3) - 2x(2x + 1) + 8x(x - 1)$

(b) $a(3a^2 - 2) - 5a^2(a + 1) - 3(a^3 - 1)$

(c) $a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)$

(d) $\frac{a}{3}(a + 2) - \frac{a}{2}(a - 1) - 2a + 3$

**2. Multiply :**

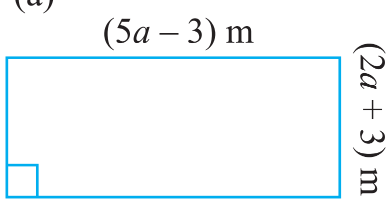
(a) $(3x - 2y)(4x + 3y)$ (b) $(2y - 1)(3 + 2y)$

(c) $(7x + 2y)(7x - 2y)$ (d) $(x - y + z)(x + y)$

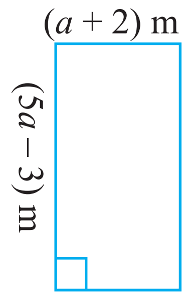
(e) $(3x + 2)(x^2 - 2x + 1)$ (f) $(x^2 - 2x)(3x^2 + 2x + 3)$

**3. Find the area of the following rectangle:**

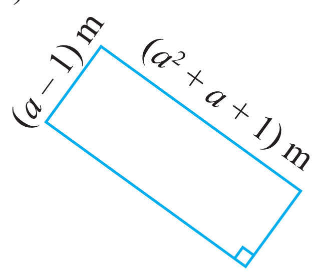
(a)



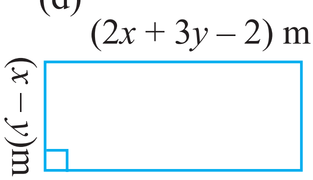

(b)



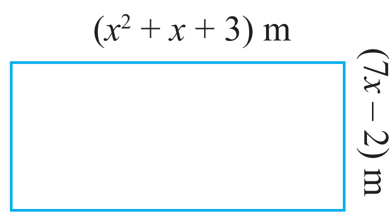

(c)




(d)




(e)

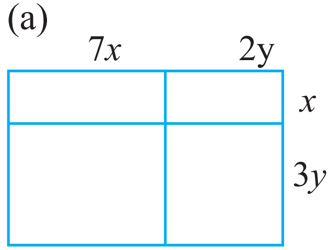



4. (a) If the length and breadth of a rectangular land are $(3x + 2)$m and $(2x - 7)$m respectively, find the area of the land.

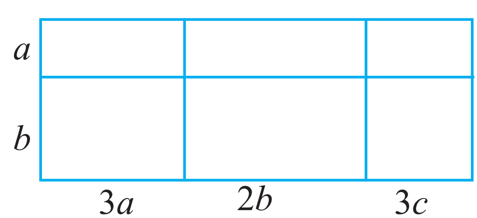
(b) If the length and breadth of a playground are $(3x - 2y)$m and $(x - y + 3)$m respectively, find the area of the playground.

**5. Find the area of the following rectangular object.**

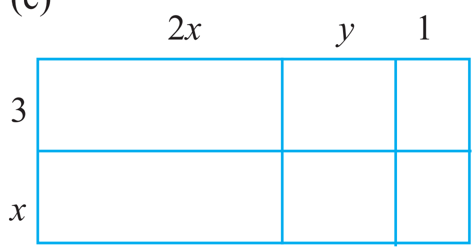
(a)



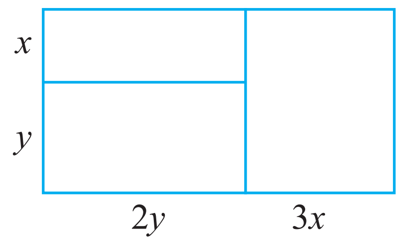

(b)



(c)




(d)




6. Find the multiple of $(3x + 2)$ and $(2x - 1)$. Find the value of it when $x = 20$.
7. Find the multiple of $(7x - 5y)$ and $(x + y - 2)$. Find the value of it when $x = 10$ and $y = 5$.

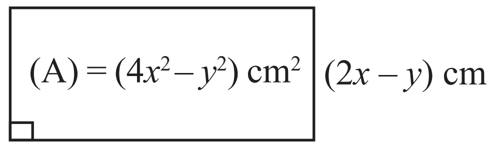
## Answer

1. (a) $7x^2 - x$ (b) $-5a^3 - 5a^2 - 2a + 3$
   (c) $0$ (d) $\frac{-a^2}{6} - \frac{5a}{6} + 3$
2. (a) $12x^2 + x - 6y^2$ (b) $4y^2 + 4y - 3$
   (c) $49x^2 - 4y^2$ (d) $x^2 - y^2 + xz + yz$
   (e) $3x^3 - 4x^2 - x + 2$ (f) $3x^4 - 4x^3 - x^2 - 6x$
3. (a) $(10a^2 + 9a - 9)\text{cm}^2$ (b) $(5a^2 + 7a - 6)\ \text{cm}^2$
   (c) $(a^3 - 1)\text{m}^2$ (d) $2x^2 + xy - 2x + 2y - 3y^2$
   (e) $(7x^3 + 5x^2 + 19x - 6)\ \text{m}^2$
4. (a) $(6x^2 - 17x - 14)\text{m}^2$ (b) $(3x^2 - 5xy + 9x - 6y + 2y^2)\ \text{m}^2$
5. (a) $(7x^2 + 23xy + 6y^2)$ (b) $3a^2 + 5ab + 3bc + 3ac + 2b^2$
   (c) $2x^2 + xy + 7x + 3y + 3$ (d) $3x^2 + 5xy + 2y^2$
6. $(6x^2 + x - 2)$, 2418
7. $7x^2 + 2xy - 14x + 10y - 5y^2)$, 585

## 11.3 Division of Binomial or Trinomial by Binomial Algebraic Expression

### Activity 1

What is the length of a rectangle whose area is $(4 \times 2 - y2)$ cm$^2$ and breadth is $(2x - y)$ cm ? Discuss.




Here , area of rectangle (A) = $(4x^2 - y^2)$ cm$^2$

breadth (b) = $(2x - y)$ cm

length ($l$) = ?

Now, discuss how we can find its length.

Since A = $l \times b$

or, $l = \dfrac{A}{b} = \dfrac{4x^2 - y^2}{2x - y}$

$$
\begin{array}{r|l}
2x - y & 4x^2 - y^2 \;(2x + y \\
 & 4x^2 - 2xy \\
 & \underline{- \quad\quad +} \\
 & 2xy - y^2 \\
 & 2xy - y^2 \\
 & \underline{- \quad\quad +} \\
 & 0
\end{array}
$$

Step 1: Multiply $2x$ by $2x$ to make $4x^2$.

Step 2: Multiply $(2x - y)$ by $2x$ and subtract.

Step 3: Multiply $2x$ by $y$ to make $2xy$.

Step 4: Multiply $(2x - y)$ by $y$ and then subtract.

**Checking,**

$(2x - y)(2x + y)$

$= 2x(2x + y) - y(2x + y)$

$= 4x^2 + 2xy - 2xy - y^2$

$= 4x^2 - y^2$

Hence, length of rectangle (l) is $(2x + y)$ cm.

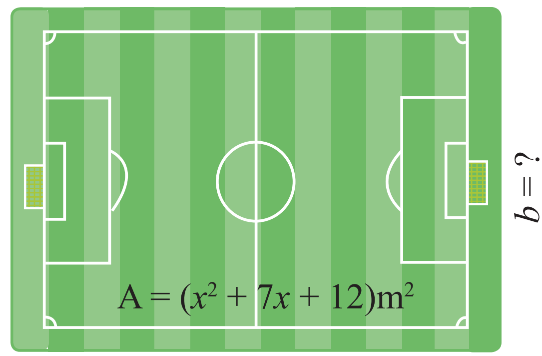
## Activity 2

The area of a rectangular playground is $(x^2 + 7x + 12)$ $m^2$ and length is $(x + 4)$ m, find the breadth. Also draw the figure.

Here, area (A) $= (x^2 + 7x + 12)m^2$

length ($l$) $= (x + 4)$ m

breadth ($b$) = ?




Now,

$$A = l \times b$$

Or, $$b = \frac{A}{l}$$

$$b = \frac{x^2 + 7x + 12}{x + 4}$$

$$
\begin{array}{r|l}
x + 4 & x^2 + 7x + 12 \; (x + 3 \\
 & -x^2 \mp 4x \\ \hline
 & 3x + 12 \\
 & -3x \mp 12 \\ \hline
 & 0
\end{array}
$$

Step 1: Multiply $x$ by $x$ to make $x^2$ and subtract.

Step 2: Multiply $x$ by 3 to make $3x$ and subtract.

**Checking,**

$(x + 3)(x + 4)$

$= x(x + 4) + 3(x + 4)$

$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

Therefore, breadth of playground is $(x + 3)$ m.

## Activity 3

Divide $(2x^2 – 3x + 7)$ by $(2x – 1)$

Here $(2x^2 – 3x + 7) \div (2x – 1)$

$$
\begin{array}{r|l}
2x - 1 & 2x^2 - 3x + 7 \; (x - 1 \\
 & \underline{\overset{-}{}2x^2 \overset{+}{-} x} \\
 & -2x + 7 \\
 & \underline{\overset{+}{-}2x \overset{-}{+} 1} \\
 & 6
\end{array}
$$

**Checking,**

$(x – 1) (2x – 1) + 6$

$= x(2x – 1) – 1(2x – 1) + 6$

$= 2x^2 – x – 2x + 1 + 6$

$= 2x^2 – 3x + 7$

∴ Therefore, Quotient = $(x –1)$, Divisor = $(2x –1)$

Dividend = $(2 \times 2 – 3x + 7)$ , Remainder = 6

**Hence, Dividend = Divisor × Quotient + Remainder.**

## Example 1

**Divide $(x^2+7x +10)$ by $(x + 2)$ and check.**

**Solution**

$(x^2 +7x + 10) \div (x + 2)$

$$
\begin{array}{r|l}
x + 2 & x^2 + 7x + 10 \; (x + 5 \\
 & \underline{\overset{-}{}x^2 \overset{-}{+} 2x} \\
 & 5x + 10 \\
 & \underline{\overset{-}{}5x \overset{-}{+} 10} \\
 & 0
\end{array}
$$

Step 1: To make $x^2$, $x$ is multiply by $x$. So, multiply $x + 2$ by $x$ and subtract from dividend.

Step 2: $x$ is multiply by 5 to make $5x$ and then subtract.

**Checking,**

$(x + 2) (x + 5)$

$= x(x + 5) + 2(x + 5)$

$= x^2 + 5x + 2x + 10$

$= x^2 + 7x + 10$

Hence, Quotient = $(x + 5)$, Divisor = $(x + 2)$ Dividend = $(x^2 + 7x +10)$

Remainder = 0

## Example 2

**Divide $(x^2 + 7x + 25)$ by $(x + 3)$ . If $x = 40$, find the real value of quotient and dividend.**

**Solution**

Here, when $(x^2 + 7x + 25)$ is divided by $(x + 3)$

$$
\begin{array}{r|l}
x + 3 & x^2 + 7x + 25 \;(x + 4 \\
 & \underline{-\; x^2 \pm 3x} \\
 & \quad 4x + 25 \\
 & \quad \underline{-\; 4x + 12} \\
 & \qquad 13
\end{array}
$$

When $x = 40$

Real value of quotient $= (x^2 + 7x + 25)$

$= 40^2 + 7 \times 40 + 25 = 1905$

Real value of dividend $= x + 4 = 40 + 4 = 44$

## Example 3

**Divide $(16x^2 + 24xy + 9y^2)$ by $(4x + 3y)$**

**Solution**

Here $(16x^2 + 24xy + 9y^2) \div (4x + 3y)$

$$
\begin{array}{r|l}
4x + 3y & 16x^2 + 24xy + 9y^2 \;(4x + 3y \\
 & \underline{-\; 16x^2 \pm 12xy} \\
 & \quad 12xy + 9y^2 \\
 & \quad \underline{-\; 12xy \pm 9y^2} \\
 & \qquad 0
\end{array}
$$

Hence, quotient $= 4x + 3y$

## Example 4

**Divide $(x^3 - y^3)$ by $(x - y)$**

**Solution**

Here $(x^3 - y^3) \div (x - y)$

$$
\begin{array}{r|l}
x - y & x^3 - y^3 \quad (x^2 + xy + y^2 \\
 & \underline{\overset{-}{x^3} \quad \overset{+}{\mp} x^2y} \\
 & x^2y - y^3 \\
 & \underline{\overset{-}{x^2y} \quad \overset{+}{\mp} xy^2} \\
 & xy^2 - y^3 \\
 & \underline{\overset{-}{xy^2} \quad \overset{+}{-} y^3} \\
 & 0
\end{array}
$$

Hence, Quotient = $(x^2 + xy + y^2)$

## Exercise 11.2

**1. Divide.**

(a) $(4a^2 + 12a) \div (2a + 6)$

(b) $(4x^2 - 9) \div (2x + 3)$

(c) $(2a^2 - 7a + 6) \div (2a - 3)$

(d) $(x^2 + 4x + 4) \div (x + 2)$

(e) $(15x^2 + 5xy - 4y^2) \div (3x + 2y)$

(f) $(35a^2 - ab - 12b^2) \div (5a - 3b)$

(g) $(x^3 - 27y^3) \div (x - 3y)$

(h) $(8x^3 + 27y^3) \div (2x + 3y)$

(i) $(6x^4 - 5x^2y^2 - 6y^4) \div (3x^2 + 2y^2)$

(j) $(a^4 - b^4) \div (a - b)$

**2. Find the unknown sides of the following rectangle.**

(a) Breadth = $(x - y)$m  Area = $(x^2 - y^2)$m$^2$

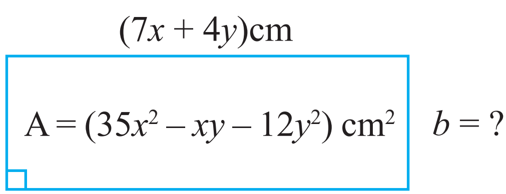
(b) Length = $(2x + 5y)$m  Area = $(2x^2 + 3xy - 5y^2)$m$^2$

(c) Breadth = $(2a - b)$m  Area = $(2a^2 + 5ab - 3b^2)$m$^2$

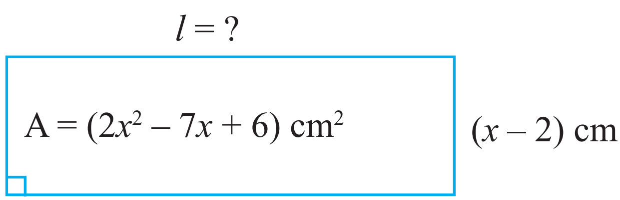
(d) Length = $(a + 3)$m  Area = $(a^3 + 27)$m$^2$

**3. Find the unknown sides in the figures given below.**

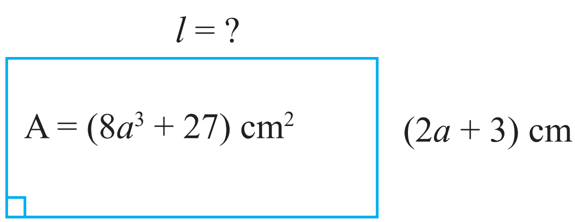
(a)



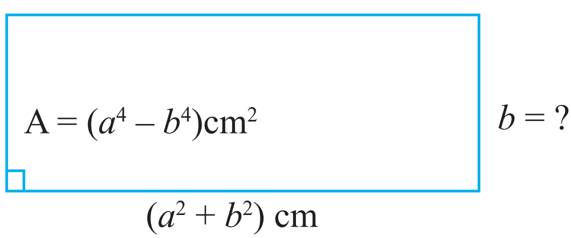

(b)




(c)




(d)




4. (a) The product of two expression is $(2a^2 + 13a + 24)$. If one expression is $(a + 8)$, find the other expression.

(b) What will be the result if $(9x^4 - 4y^4)$ is divided by $(3x^2 - 2y^2)$ ?

**5. The area of floor of a room is $(15x^2 + 4xy - 4y^2)$ m². If the breadth of this room is $(5x - 2y)$m, find:**

(a) The length of room.

(b) If $x=1$ and $y=2$, find the actual length , breadth and area of the floor of room.

6. What will come if $(x^2 + 19x + 54)$ is divided by $(x + 3)$? If $x=1$, find the actual value of dividend, divisor and quotient.

**7. Test the relationship among Quotient, Divisor, Dividend and Remainder.**

(a) $(a^2 + 7a + 13) \div (a + 3)$

(b) $(2a^2 - 5a + 23) \div (2a - 3)$

## Answer

1. (a) $2a$ (b) $(2x - 3)$ (c) $(a - 2)$

(d) $(x + 2)$ (e) $(5x - 2y)$ (f) $(7a + 4b)$

(i) $x^2 + 3xy + 9y^2$ (j) $4x^2 - 6xy + 9y^2$

(k) $(2x^2 - 3y^2)$ ($l$) $a^3 + a^2b + ab^2 + b^3$

2. (a) $(x + y)$ m (b) $(x - y)$ m

(c) $(a + 3b)$ m (d) $(a^2 - 3a + 9)$ m

3. (a) $(5x - 3y)$ cm (b) $(2x - 3)$ cm (c) $(4a^2 - 6a + 9)$ cm

(d) $(a^2 - b^2)$ cm 4. (a) $(2a - 3)$ (b) $(3x^2 + 2y^2)$

5. (a) $(3x + 2y)$ m (b) 7 m, 1 m, 7 m²

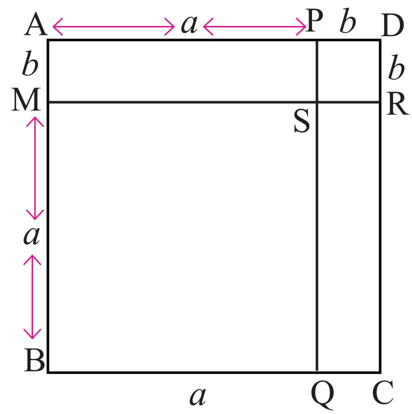
6. Quotient = $x + 16$ the actual value of dividend, divisor and quotient is 74, 4, 17 respectively.

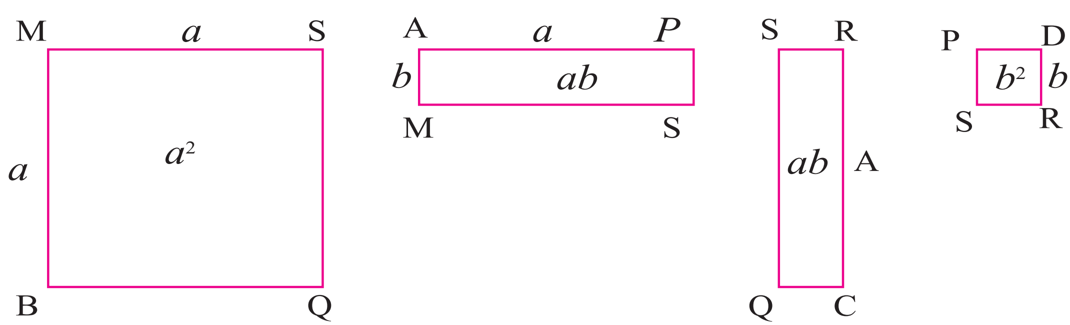
7. Show the answers to your teacher.

## 11.4 Geometrical Concept and Application of $(a + b)^2$

### Activity 1

Draw a square ABCD on a chart paper which is shown in the figure below. From the point A, take a unit length on AD and b unit length on AB and mark them. In the same way, mark as PD = SR = QC = b. Now cut PQ and MR and find its area.




Is the area of sum of small pieces of paper equal to the area of whole figure? Discuss in the class.

Now Area of Square ABCD = Area of (Square MBQS + Rectangle AMSP + Rectangle SRCQ + Square PSRD).

or, $(a + b)^2 = a^2 + ab + ab + b^2$

Therefore, $(a + b)^2 = a^2 + 2ab + b^2$

While multiplying

$(a + b)^2 = (a + b) \times (a + b)$

$= a(a + b) + b(a + b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

## 11.5 Geometrical Concept of $(a - b)^2$

**Activity 3**

How can we find the area of square given in the figure having length (a – b)? Discuss in group.

Area of square PQRS = $a \times a = a^2$

Area of square BCDS = $b \times b = b^2$

Area of rectangle = PD × PA

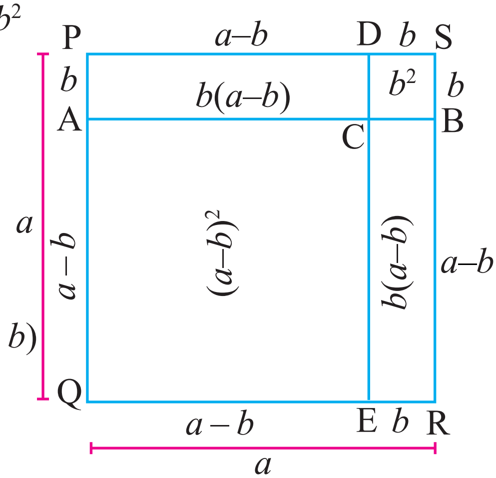
$= (a - b) \times b = ab - b^2$

Area of rectangle CBRE = BR × ER

$= (a - b) \times b = ab - b^2$

Area of square AQEC = $(a - b) \times (a - b)$

$= (a - b)^2$




Now, Area of AQEC

= Area of (Square PQRS – Rectangle PACD – Square BCDS – Rectangle CBRE)

or, $(a - b)^2 = a^2 - (ab - b^2) - b^2 - (ab - b^2)$

or, $(a - b)^2 = a^2 - ab + b^2 - b^2 - ab + b^2$

or, $(a - b)^2 = a^2 - 2ab + b^2$

Hence, $(a - b)^2 = a^2 - 2ab + b^2$

**Formulae**

(a) $(a - b)^2 = a^2 - 2ab + b^2$

(b) $(a + b)^2 = a^2 + 2ab + b^2$

## Example 1

**Write $(x + 2)^2$ in expanded form.**

(a) Without using formula

(b) Using formula

(c) With geometrical figure

### Solution

Here,

(a) The square of $(x + 2) = (x + 2)^2$

$= (x + 2)\ (x + 2)$

$= x\ (x + 2) + 2\ (x + 2)$

$= x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$

(b) The square of $(x + 2) = (x + 2)^2$

$= x^2 + 2 \times x \times 2 + 2^2$

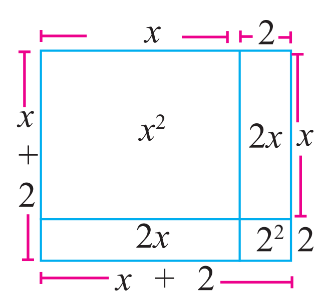
$= x^2 + 4x + 4$

$[\because (a + b)^2 = a^2 + 2ab + b^2]$

(c) The square of $(x + 2)$

or, $(x + 2)^2 = x^2 + 2x + 2x + 2^2$

$\therefore (x + 2)^2 = x^2 + 4x + 4$



## Example 2

**Find the square of the following number using $(a + b)^2$ and $(a - b)^2$.**

(a) 12 (b) 88

**Solution**

Here,

(a) 12

$= (10 + 2)^2$

$= 10^2 + 2 \times 10 \times 2 + 2^2$

$= 100 + 40 + 4$

$= 144$

(b) 88

$= (90 - 2)^2$

$= 90^2 - 2 \times 90 \times 2 + 2^2$

$= 8100 - 360 + 4$

$= 7744$

## Example 3

**Find the square of the following expression.**

(a) $\left(2x - \frac{1}{3x}\right)$ (b) $\left(3x + \frac{1}{3}\right)$ (c) $(a - b + c)$ (d) $(x + y + z)$

**Solution**

Here,

(a) The square of $\left(2x - \frac{1}{3x}\right) = \left(2x - \frac{1}{3x}\right)^2$

$= (2x)^2 - 2 \times 2x \times \frac{1}{3x} + \left(\frac{1}{3x}\right)^2$

$= 4x^2 - \frac{4}{3} + \frac{1}{9x^2}$

(b) The square of $\left(3x + \frac{1}{3}\right)$

$= \left(3x + \frac{1}{3}\right)^2$

$= (3x)^2 + 2 \times 3x \times \frac{1}{3} + \left(\frac{1}{3}\right)^2$

$= 9x^2 + 2x + \frac{1}{9}$

(c) The square of $(a - b + c) = (a - b + c)^2$

$= (a - b)^2 + 2(a - b)c + c^2$

$= a^2 - 2ab + b^2 + 2ac - 2bc + c^2$

(d) The square of $(x + y + z) = (x + y + z)^2$

$= (x + y)^2 + 2(x + y)z + z^2$

$= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$

$= x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$

## Example 4

**Simplify : $(2a + b)^2 - (2a - b)^2$**

**Solution**

Here $(2a + b)^2 - (2a - b)^2$

$= (2a)^2 + 2 \times 2a \times b + b^2 - \{(2a)^2 - 2 \times 2a \times b + b^2\}$

$= 4a^2 + 4ab + b^2 - 4a^2 + 4ab - b^2$

$= 8ab$

## Example 5

**If $x + \frac{1}{x} = 6$ find the value of the following.**

(a) $\left(x^2 + \frac{1}{x^2}\right)$ (b) $\left(x - \frac{1}{x}\right)^2$

**Solution**

Here,

(a) $\left(x + \frac{1}{x}\right) = 6$

or, $\left(x + \frac{1}{x}\right)^2 = 6^2$ [∵ Squaring on the both sides ]

or, $x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 36$ [∵ $(a + b)^2 = a^2 + 2ab + b^2$]

or, $x^2 + 2 + \left(\frac{1}{x^2}\right) = 36$

or, $x^2 + \frac{1}{x^2} = 36 - 2$

or, $x^2 + \frac{1}{x^2} = 34$

(b) $\left(x + \frac{1}{x}\right) = 6$

or, $\left(x + \frac{1}{x}\right)^2 = 6^2$ [∵ Squaring on the both sides ]

or, $\left(x - \frac{1}{x}\right)^2 + 4 \times x \times \frac{1}{x} = 36$ [∵ $(a + b)^2 = (a - b)^2 + 4ab$]

or, $\left(x - \frac{1}{x}\right)^2 = 36 - 4$

or, $\left(x - \frac{1}{x}\right)^2 = 32$

## Example 6

**If $(a + b) = 8$, $ab = 12$, find the value of :**

(a) $a^2 + b^2$ (b) $(a - b)$

**Solution**

Here,

(a) $a + b = 8$

or, $(a + b)^2 = 8^2$ [∵ Squaring on the both sides ]

or, $a^2 + 2ab + b^2 = 64$

or, $a^2 + b^2 = 64 - 2ab$

$= 64 - 2 \times 12$ [∵ $ab = 12$]

$= 64 - 24$

$= 40$

Hence, $a^2 + b^2 = 40$

(b) $(a + b) = 8$

or, $(a + b)^2 = 8^2$ [$\because$ Squaring on the both sides ]

or, $(a - b)^2 + 4ab = 64$

or, $(a - b)^2 + 4 \times 12 = 64$

or, $(a - b)^2 = 64 - 48 = 16$

or, $(a - b)^2 = 4^2$

Hence, $(a - b) = 4$

## Example 7

**If $a - \frac{1}{a} = 15$, prove that**

(a) $\left(a^2 + \frac{1}{a^2}\right) = 227$ (b) $\left(a + \frac{1}{a}\right)^2 = 229$

**Solution**

Here,

(a) $\left(a - \frac{1}{a}\right) = 15$

or, $\left(a - \frac{1}{a}\right)^2 = 15^2$ [$\because$ Squaring on the both sides ]

or, $a^2 - 2 \times a \times \frac{1}{a} + \left(\frac{1}{a}\right)^2 = 225$

or, $a^2 - 2 + \frac{1}{a^2} = 225$

or, $a^2 + \frac{1}{a^2} = 225 + 2$

Hence, $a^2 + \frac{1}{a^2} = 227$ Proved.

(b) $\left(a - \frac{1}{a}\right) = 15$

or, $\left(a - \frac{1}{a}\right)^2 = 15^2$ [$\because$ Squaring on the both sides ]

or, $\left(a + \frac{1}{a}\right)^2 - 4 \times a \times \frac{1}{a} = 225$ [$\because (a - b)^2 = (a + b)^2 - 4ab$]

or, $\left(a + \frac{1}{a}\right)^2 = 225 + 4$

Hence, $\left(a + \frac{1}{a}\right)^2 = 229$

## Exercise 11.3

**1. Find the square of the following expression (a) by using formula (b) Without using formula (c) by geometrical figure.**

(a) $(x + 3)$ (b) $(x - 1)$ (c) $(a + 4)$ (b) $(a - 5)$

**2. Find the square of the following expression.**

(a) $(3x^2 + 2)$ (b) $(5x - 2y)$ (c) $\left(3x^2 - \frac{1}{3y}\right)$

(d) $\left(x^2 + \frac{1}{2x}\right)$ (e) $(x - y + z)$ (f) $(x^2 + y^2 + z^2)$

**3. Find the square of the following expression.**

(a) 98 (b) 102 (c) 999

**4. Write the following expression in whole square form.**

(a) $x^2 - 2 + \frac{1}{x^2}$ (b) $4x^2 - 20xy + 25y^2$

(c) $9x^2 + 12xy + 4y^2$ (d) $81a^4 + 72a^2b^2 + 16b^4$

(e) $a^2b^2 + \frac{10ab}{xy} + \frac{25}{x^2y^2}$

**5. Simplify.**

(a) $(2c - 5d)^2 - (5d - 2c)^2$ (b) $(3x - 2y)^2 + (3y - 2x)^2$

(c) $\left(x - \frac{1}{x}\right)^2 - \left(x + \frac{1}{x}\right)^2$ (d) $\left(c + \frac{1}{d}\right)^2 - \left(c - \frac{1}{d}\right)^2$

**6. If $a + \frac{1}{a} = 10$, find the value of**

(a) $a^2 + \frac{1}{a^2}$ (b) $\left(a - \frac{1}{a}\right)^2$

**7. If $m - \frac{1}{m} = 6$, find the value of**

(a) $m^2 + \frac{1}{m^2}$ (b) $\left(m + \frac{1}{m}\right)^2$

**8. If $(x + y) = 9$, $xy = 8$, then find the value of**

(a) $x^2 + y^2$ (b) $(x - y)$

9. If $a^2 + b^2 = 17$ and $ab = 4$ and $ab = 4$ then, find the value of $(a + b)$.

10. If $p^2 + \frac{1}{p^2} = 7$, find the value of $p - \frac{1}{p}$

## Project Work

Use carton board and scissers to prepare a model to prove show the derivation of formula of $(a + b)^2$. Present the model in the class.

## Answer

1. (a) $x^2 + 6x + 9$ (b) $x^2 - 2x + 1$
   (c) $a^2 + 8a + 16$ (d) $a^2 - 10a + 25$
2. (a) $9x^4 + 12x^2 + 4$ (b) $25x^2 - 20xy + 4y^2$
   (c) $9x^4 - \frac{2x^2}{y} + \frac{1}{9y^2}$ (d) $x^4 + x + \frac{1}{4x^2}$
   (e) $x^2 + y^2 + z^2 - 2xy - 2yz + 2zx$
   (f) $x^4 + y^4 + z^4 + 2x^2y^2 + 2y^2z^2 + 2z^2x^2$
3. (a) 9604 (b) 10404 (c) 998001
4. (a) $(x - \frac{1}{x})^2$ (b) $(2x - 5y)^2$ (c) $(3x + 2y)^2$
   (d) $(9a^2 + 4b^2)^2$ (e) $(ab + \frac{5}{xy})$
5. (a) 0 (b) $13x^2 - 24xy + 13y^2$
   (c) $-4$ (d) $\frac{4c}{d}$
6. (a) 98 (b) 96
7. (a) 38 (b) 40
8. (a) 65 (b) 7
9. +5
10. +3

# Lesson 12 Equation, Inequality and Graph

## 12.0 Review

**1. Read the following question, discuss and answer them.**

(a ) Is the sum of 5 and 7 is 11?

(b) If $x - 3 = 5$, what is the value of $x$?

(c) If $3a = 18$, what should be the value of a?

(d) If $\frac{3x}{5} = 12$, what is the value of $x$?

(e) If the sum of $x$ and 10 is 18, what is the value of $x$?

(f) Is $-10 > -15$ true?

(g) If $x - 4 < 8$, what is the solution set of $x$?

## 12.1 Probem of Linear Equation on two variables

### Activity 1

Hari and Gita had gone to the market to buy vegetable. Hari bought $x$ kg potatoes and Gita bought $y$ kg onion. If the total weight of potatoes and onion is 30 kg, discuss the following questions.

(a) How can you write the total quantity of potatoes and onion which they bought in an equation?

(b) If potatoes is 20 kg, how much is the onion?

30 kg

| Potato $x$ kg | Onion $y$ kg |
|---|---|
| $x + y = 30$ | |

30 kg

| Potato $x$ kg = 20kg | Onion $y$ kg |
|---|---|
| Onion $y = (30 - 20) = 10$ kg | |

Here, $x + y = 30$ kg

If the quantity of potato is 20 kg, we can find the quantity of onion in the following way:

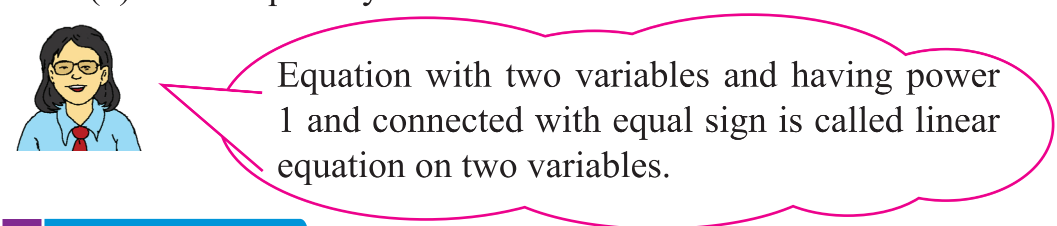
$y = 30 - 20 = 10$ kg.

## Activity 2

A school has decided to fence its compound by wire. The length and breadth of school is $x$ m and y m respectively. If 300 m wire is needed to fence it once,

(a) How can you make the equation? Discuss in class.

(b) Which quantity is variable and which is constant?




## Example 1

**If the sum of length and breadth of a room is 22 m:**

(a) Write in equation form.

(b) If the length is 15 m, find the breadth of room.

### Solution

Suppose length of room = $x$ m

Breadth of room = $y$ m

(a) According to question $x + y = 22$

(b) When $x = 15$,

$x + y = 22$

or, $15 + y = 22$

or, $y = 22 - 15$

or, $y = 7$ m

Therefore, breadth of room is 7 m.

## Example 2

**If Ram has 17 pens of two colours red and black, then**

(a) Write the equation with two variables denoting by two colors.

(b) If the number of black pen is 5, find the number of red pens.

**Solution**

Suppose, number of red pens = $x$

Number of black pens = $y$

Total number of pens = 17

(a) Now, $x + y = 17$

$x + y = 17$

Therefore, required equation is $x + y = 17$

(b) If number of black pen ($x$) = 5

Then $x + y = 17$

or, $5 + y = 17$

or, $y = 17 - 5$

or, $y = 12$

**Checking,**

x + y = 17

or, 5 + 12 = 17

or, 17 = 17

or, $y = 12$.

Therefore, number of black pen = 12.

## Example 3

**If the sum of ages of Rama's father and two times of her ages is 60,**

(a) Find the equation which represent the sum of their ages.

(b) If Rama is 10 years old, find the age of her father.

**Solution**

Suppose age of Rama = $x$ years

Age of her father = $y$ years

(a) According to question, $2x + y = 60$

Therefore, the required equation is $2x + y = 60$

(b) If Rama is 10 years old ,

Fathers age = ?

Now putting $x = 10$ in equation $2x + y = 60$

$2 \times 10 + y = 60$

or, $20 + y = 60$

or, $y = 60 - 20$

or, $y = 40$

Therefore, if Rama is 10 years old, then her father is 40 years.

## Exercise 12.1

**1. Write if the following sentences are true or false.**

(a) $x + 4 = 8$ is an equation of two variables.

(b) $2x = 3y$ is an equation of two variables.

(c) Equation with two variable and with equality sign is called equation with two variables.

(d) $3x = 12$ is an equation of two variables.

**2. If a land has the length *x* cm and breadth *y* cm. If the perimeter of land is 240 cm, find:**

(a) The equation that represent the perimeter.

(b) If the length of land is 70 cm, find it's breadth.

**3. A 5 m path is around the rectangular pond. If the perimeter including path is 210 m find:**

(a) The equation that represents length and breadth.

(b) If the breadth of the pond is 40 m, find it's length.

4. If the sum of two consecutive odd numbers is 56, find the numbers.

5. There are 75 students in a class. If the number of boys is double of the number of girls, find the number of boys and girls.

6. The sum of present age of Subhasa and Supreme is 40 years. If the age of Subhasa is three times the age of Supreme, find their present ages.

**7. If there are 42 students in a class:**

(a)Write the equation which represent all students.

(b) What are the possible number of boys and girls in the class?

## Answer

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1. | (a) | wrong | (b) | 17s | (c) | 17s | (d) | wrong |
| 2. | (a) | $x + y = 120$ | (b) | 50 m | | | | |
| 3. | (a) | $x + y = 85$ | (b) | 45 m | | | | |
| 4. | 27 and 29 | | 5. | 50 and 25 | | | | |
| 6. | 30 years and 10 years | | | | | | | |
| 7. | Show the answers to your teacher. | | | | | | | |

## 12.3 Representation of Inequality in the number line

**Activity 1**

**If $x$ and $y$ are the two whole number, how can we express these number in mathematical sentence? Discuss in class.**

(a) Is $x$ and $y$ are equal?

(b) Is $x < y$

(c) Is $x > y$

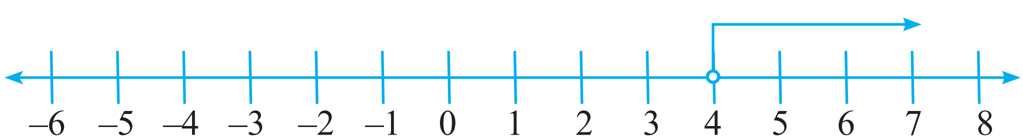
To know the answer of above questions, take whole number in place of $x$ and $y$. For example, if we take the numbers greater than 4 like $4 < 5$, $4 < 6$, $4 < 7$, $4 < 8$, then then we write $x > 4$.

**Activity 2**

**How can we represent $x > 4$ in number line? Discuss in class.**

Here the values of $x$ are only the the number greater than 4. So the solution set of x are $x = \{5, 6, 7, \ldots\ldots\ldots\}$.

Representation in number line are as below.




*4 is not in the solution set of x, so in number line, it is denoted by (o).*

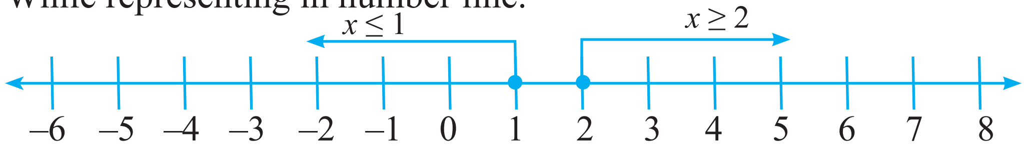
**Activity 3**

**Discuss in group and show $x \geq 2$ and $x \geq 3$ in number line.**

Here $x \geq 2$ the solution set of $x = \{2, 3, 4, ...\}$

$x \leq 1$ The solution set of $x = \{1, 0, -1, -2, ...\}$

While representing in number line.




( The number 2 lies in solution set $x \geq 2$ and the number 1 lies in solution set $x \leq 1$. So in number line it is denoted (●).)

## Some rules of inequality

If $x$ and $y$ are the two whole number in which $x > y$ and $z$ is another whole number then

(a) $x + z > y + z$ (addition axiom)

(b) $x - z > y - z$ (subtraction axiom)

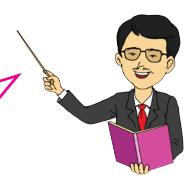
(c) $xz > yz$ (multiplication axiom)

(d) $\frac{x}{z} > \frac{y}{z}$ (division axiom z is positive)

(e) $xz < yz$ (multiplication axiom )

(f) $\frac{x}{z} > \frac{y}{z}$ (division axiom z is negative)

(In mathematical sentences including inequality ($<, \leq, >, \geq$) if we multiply or divide by minus sign the inequality sign will be changed.)



**Let's play a game!**

(a) Make two groups with 7 students in each.
(b) Write number –3, –2, –1, 0, 1, 2, 3 in paper and stick on the chest.
(c) Sit in two benches facing the group each other.
(d) Stand a person from a group when another group ask the inequality.
For example, the number 3 stands up when $x > 2$.
(e) Play the game in this way and provide number for the right answer.
(f) When each will get a turn, the team who scores high will be the winner.

## Example 1

**Solve the inequality $x + 2 > 5$ and show in the number line.**

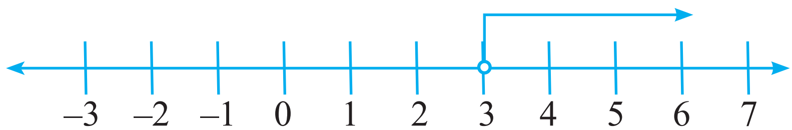
**Solution**

Here $x + 2 > 5$

or, $x + 2 - 2 > 5 - 2$ [subtracting 2 on both sides]

or, $x > 3$

While showing in number line,




The possible solution set $x = \{4, 5, 6, ...\}$

## Example 2

**Solve $3x - 2 \leq -11$ and show in the number line.**

**Solution**

Here, $3x - 2 \leq -11$

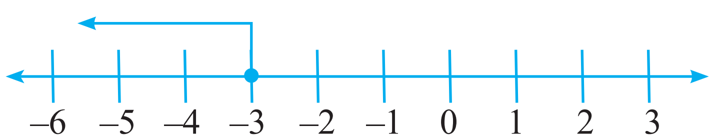
or, $3x - 2 + 2 \leq -11 + 2$ [Adding 2 on both sides]

or, $3x \leq -9$

or, $\frac{3x}{3} \leq -\frac{9}{3}$ [Dividing both sides by 3]

or, $x \leq -3$

While showing in number line,




Hence, possible solution set $x = \{-3, -4, -5, ...\}$

## Example 3

**Solve $3 - 2x \leq 9$ and show in the number line.**

**Solution**

Here $3 - 2x \leq 9$

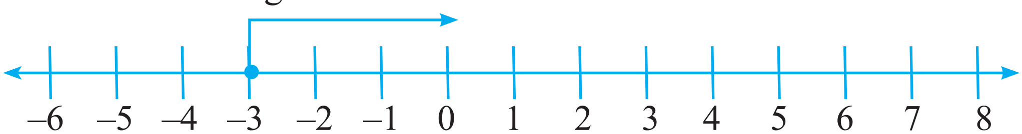
or, $-3 + 3 - 2x \leq 9 - 3$ [Subtracting 3 from both sides]

or, $-2x \leq 6$

or, $\frac{-2x}{-2} \geq \frac{6}{-2}$

or, $x \geq -3$

While showing in number line

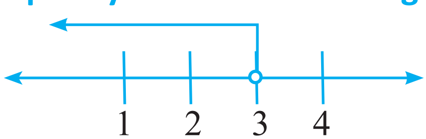



Hence, possible solution set $x = \{-3, -2, -1, ...\}$

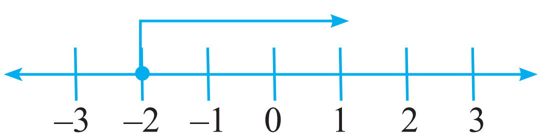
## Example 4

**Write the inequality from the following number line.**

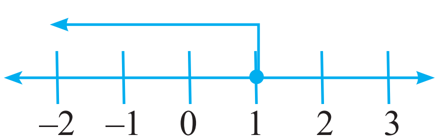
(a)



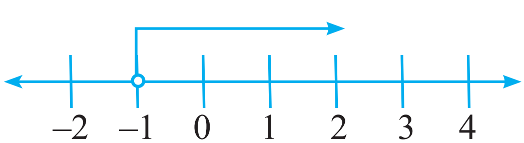

(b)




(c)




(d)



## Solution

Here,

(a) 3 is only circled and its arrow is towards the left side. so $x < 3$.

(b) –2 is circled with colour and its arrow is towards the right side. So $x \geq -2$.

(c) 1 is circled with colour and it's arrow is towards left side. So $x \leq 1$

(d) –1 is only circled and its arrow is towards right side.

So $x > -1$

### Exercise 12.3

**1. If 1, 2 and –3 are whole number, are the following true according to Trichotomy?**

(a) $1 + (-3) = 2 + (-3)$ (b) $(2) -3 < 1 + (2)$

(c) $2 - (1) > -3 - (1)$ (d) $(1) -3 < (1) + 2$

(e) $\frac{1}{-3} > \frac{2}{-3}$ (f) $(-3) \times 1 \geq 2 \times (-3)$

**2. Solve the following inequality and show in number line.**

(a) $x + 1 > 2$ (b) $x - 3 \leq 4$ (c) $x + 2 < 4$

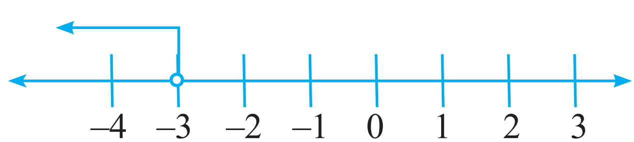
(d) $x - 2 \leq 3$ (e) $3 - x \geq 1$ (f) $4 - 2x \geq 6$

(g) $3x + 2 \geq x - 6$ (h) $5x - 3 \geq 12$ (i) $7x - 4 \leq 17$

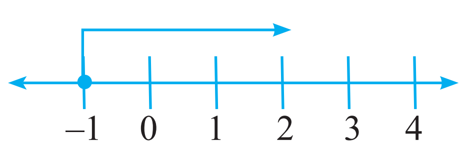
(i) $5x - 7 \leq 2x + 5$

**3. Write the inequality according to the following number line.**

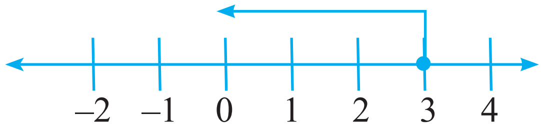
(a)



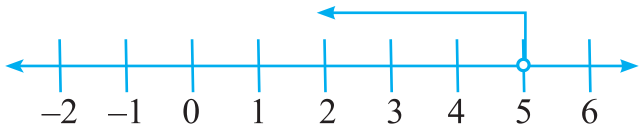

(b)



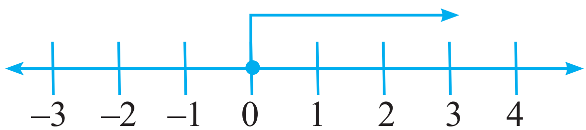
(c)



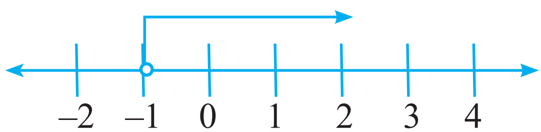

(d)




(e)




(f)




**4. Solve the following problems and show in number line.**

(a) If one fourth of a number is subtracted from 3, the result is greater than or equal to 2. Write the inequality, solve it and show in number line.

(b) If 13 is subtracted from double of a number, the result is less than or equal to 3. Write the inequality, solve it and show in number line.

(c) If 9 is subtracted from four times of a number, the result is less than or equal to –3. Write the inequality, solve and show in number line.

(d) If 7 is added to the three times of a number, the result is less than 13. Solve this in equality and show in number line.

**Answer**

Show the answers to your teacher.

## 12.4 Graph of Linear Equation in Two Variable

### Activity 1

The head teacher of a school bought some volleyball and foot ball.If she has bought 11 balls in total,

(a) Write this sentences in mathematical sentences.

(b) Take at least 3 values in variable in above equation.

(c) Show all the values of variable in graph paper.

Now discuss about the calculation of variables in your class.

After discussion check whether it is same with the following or not.

(a) Suppose number of volleyball = $x$

Number of football = $y$

Total number of balls = 11

According to question,

While writing in equation $x + y = 11$.

(b) Find the values of variables from the equation $x + y = 11$.

$x + y = 11$

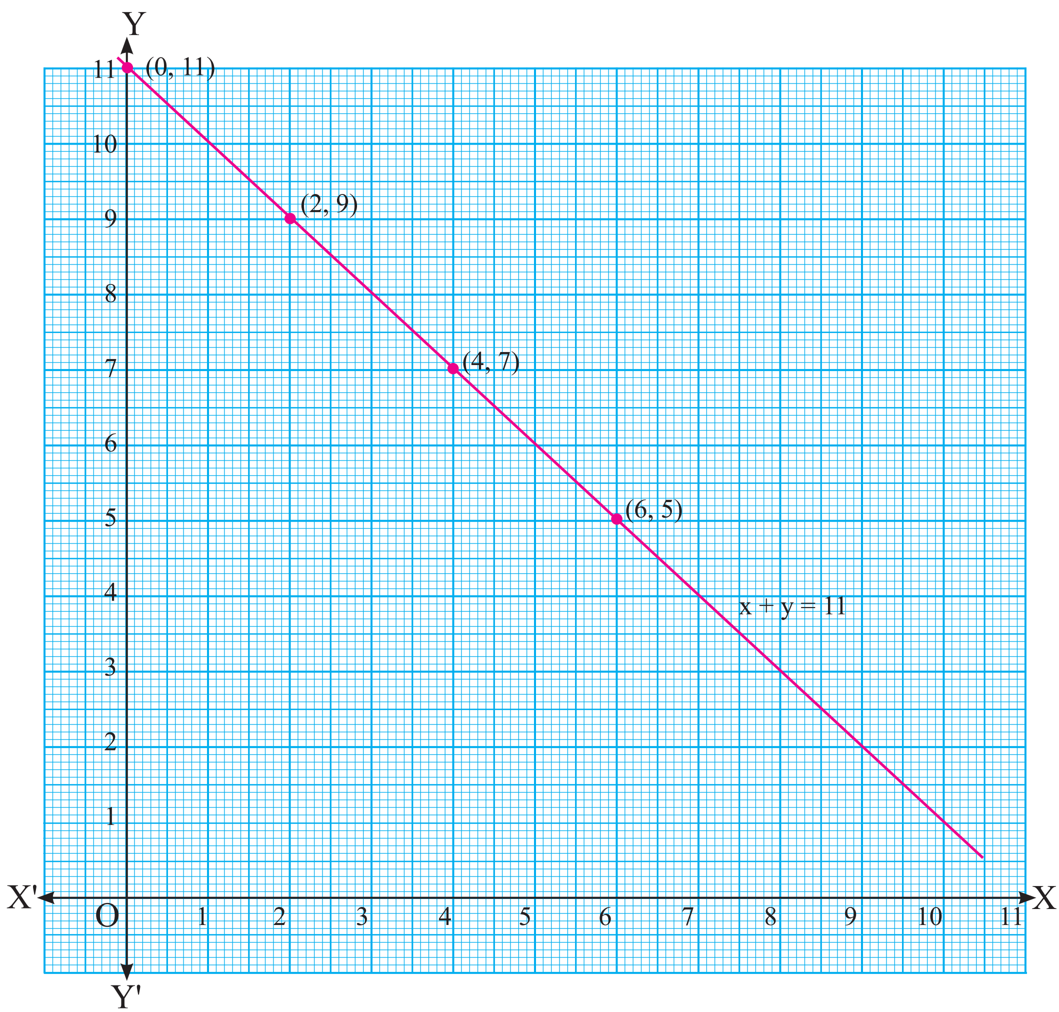
$y = 11 - x$ ............ (i)

From the equation (i) put the different values of $x$ and find $y$

| $x$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $y$ | 11 | 9 | 7 | 5 |

(c) From the above table plot the order pairs, (0, 11), (2, 9) and (6, 5) in graph.

All the points lies in same straight line.



## Example 1

**Fill the table with the value of $x$ in from the equation $y = \frac{3x-1}{2}$.**

| $x$ | –1 | 1 | 3 | –3 | 5 | –5 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

### Solution

Here, the given equation is $y = \frac{3x-1}{2}$

Finding the value of $y$ after putting the value of $x$ we get,

| $x$ | –1 | 1 | 3 | –3 | 5 | –5 |
|---|---|---|---|---|---|---|
| $y$ | –2 | 1 | 4 | –5 | 7 | –8 |

## Example 2

**Show the equation of two variable $3x + y = 6$ in graph paper.**

### Solution

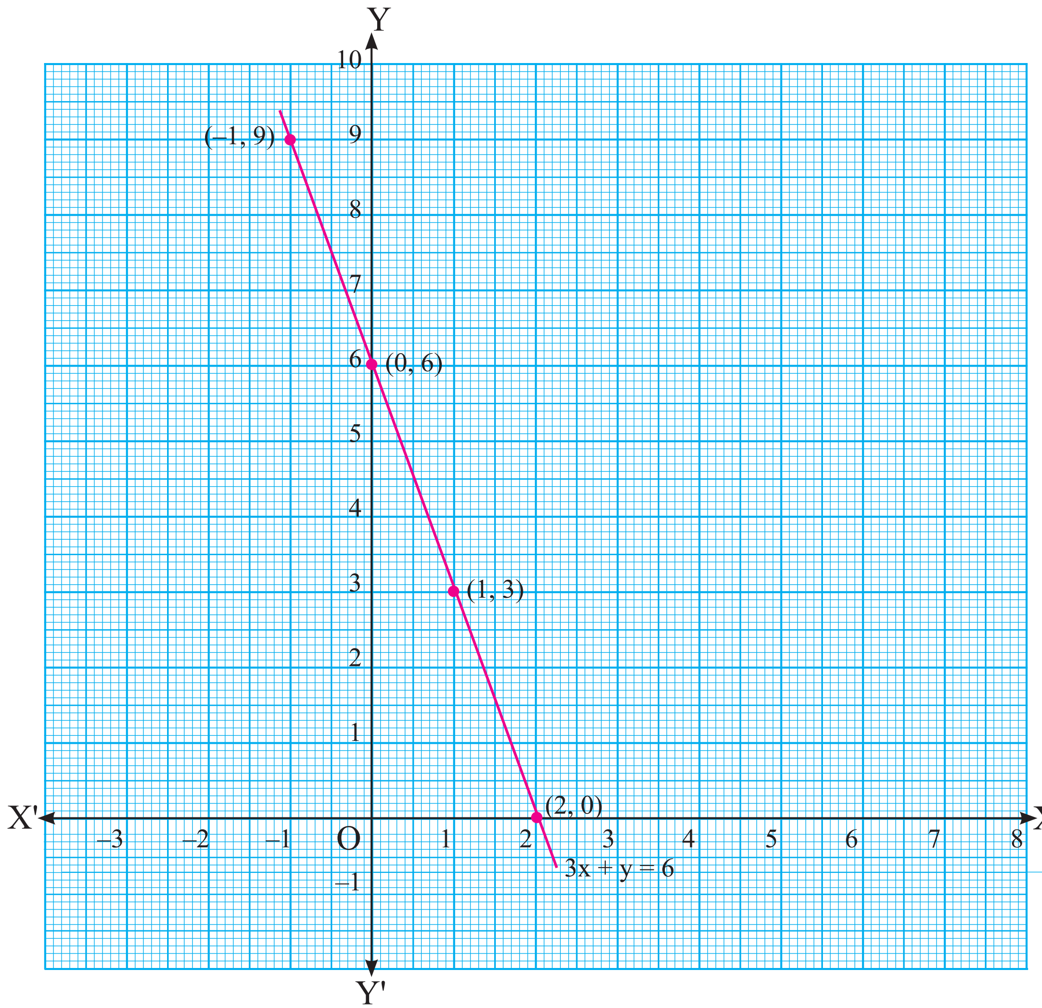
$3x + y = 6$

or, $y = 6 - 3x$ ... (i)

Now, finding the value of $y$ by putting the value of $x$, we get the following table.

| $x$ | 0 | 1 | –1 | 2 |
|---|---|---|---|---|
| $y$ | 6 | 3 | 9 | 0 |

Now show the order pairs (0, 6), (1, 3), (–1, 9), (2, 0) in graph paper.




## Exercise 12.2

**1. From the following equation, make the table for the values of *x* and *y*.**

(a) $3x + y = 6$ (b) $x + y = 4$ (c) $3x - y = 7$

**2. Draw the line graph from the following equation.**

(a) $2x + y = 8$ (b) $3x + 2y = 6$ (c) $x + y + 3 = 0$

(d) $x + 2y = 10$ (e) $y = 4x - 1$ (f) $2x + y - 3 = 0$

### Answer

Show the answers to your teacher.

## Miscellaneous exercise

**1. Convert the following multiple actions into indices (power).**

(a) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ (b) $\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}$

**2. If a = 1, b =2, c = –3, find the value of the following.**

(a) $(xyz)^{a+b+c}$ (b) $(a+b+c)^{100}$ (c) $\frac{x^{c}}{x^{-a} \times x^{-b}}$ (d) $(z)^{a} \times (z)^{b} \times (z)^{c}$

**3. If $a = 3$, $b = 1$ and $c = -4$, find the value of the following.**

(a) $2a^2 + 3b - 4bc$ (b) $a^2 + b^2 - c^2$

(c) $\frac{a+b}{c}$ (d) $\frac{a+b-c}{2ab}$

**4. Divide:**

(a) $(x^2 - 7x + 12) \div (x - 3)$ (b) $(x^4 - 81y^4) \div (x^2 - 9y^2)$

(c) $(15x^2 + 11x - 12) \div (3x + 4)$

5. If a cricket stadium has the length $(2x + y)$ m and breadth $(x - 2y)$ m, find the area of the stadium.

**6. The area of floor of a seminar hall is $(2 \times 2 - 7x + 6)$ sq.m. If is carpeted by a carpet with breadth $(x - y)$ m, find**

(a) How much length of carpet is needed to cover it?

(b) If $x = 5$, find its length, breadth and actual area .

(c) Find the cost of carpet if the cost per meter square is Rs. 400.

(d) When the value of $x$ is increased 1 from 5, how much percent is increased in cost ?

7. When $(56x^2 + 106x - 30)$ is divided by $(7x + 15)$, find the remainder.

**8. Find the area of the following by using formula, without using formula and in geometrical figure.**

(a) $(y + 4)$ (b) $(y + 5)$ (c) $(b - 2)$

9. Find the square of the following number by addition and subtraction of two numbers.

(a) 28 (b) 296 (c) 502

**10. Simplify :**

(a) $(2a - 3b)^2 - (2a + 3b)^2$ (b) $(\frac{1}{a} + a)^2 + (a - \frac{1}{a})^2$

(c) $(m^2 + n^2)^2 - (m^2 - n^2)^2$

**11. If $p + \frac{1}{p} = 12$ find the value of the following.**

(a) $p^2 + \frac{1}{p^2}$ (b) $(p - \frac{1}{p})^2$

**12. If $x + y = 15$ and $xy = 8$, find the value of:**

(a) $x^2 + y^2$ (b) $(x - y)^2$

**13. If $b - \frac{1}{b} = 8$ find the value of:**

(a) $b^2 + \frac{1}{b^2}$ (b) $(b - \frac{1}{b})^2$

**14. Make the table for $x$ and $y$ from the following equation.**

(a) $x + y = 2$ (b) $2x - y = 4$

15. **Draw the line graph from the following equations.**

(a) $2x + y = 5$ (b) $4x + 3y = 6$ (d) $3x - 2y = 7$

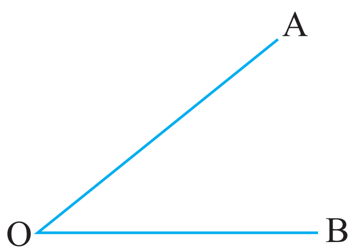
## Answer

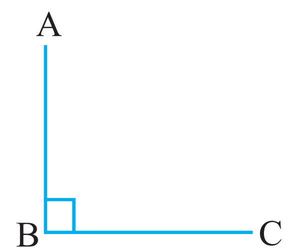
1. (a) $(\frac{2}{3})^5$ (b) $(\frac{a}{b})^6$ 2. (a) 1 (b) 0 (c) 1
(d) 1 3. (a) 37 (b) –6 (c) –1 (d) $\frac{4}{3}$
4. (a) $(x - 4)$ (b) $x^2 + 9y^2$ (c) $(5x - 3)$
5. (a) $(2x^2 - 3xy - 2y^2)\ m^2$ (b) 140 m, 1000 $m^2$
6. (a) $(2x - 3)$ m (b) 7 m, 3 m, 21 $m^2$ (c) Rs. 8400
(d) 71.43% 7. $(8x - 2)$ 8. (a) $y^2 + 8y + 16$
(b) $y^2 + 10y + 25$ (c) $b^2 - 4b + 4$ 9. (a) 784 (b) 87616
(c) 252004 10. (a) $-24ab$ (b) $2\,(a^2 + \frac{1}{a^2})$ (c) $4m^2n^2$
11. (a) 142 (b) 140 12. (a) 209 (b) 191 13. (a) 66
(b) 64 14. and 15. Show the answers to your teacher.

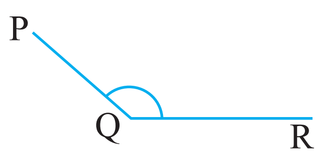
# Lines and Angles

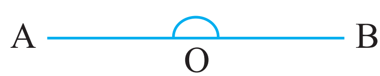
## 13.0 Review

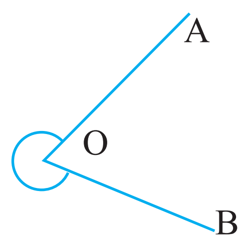
Measure the following angles and distinguish them as acute angle, right angle, obtuse angle, straight angle and reflect angle.




Measure the following angles by using compass, setsquare and protractor.

(a) 65° (b) 110° (c) 90°

Draw the following angles using protractor.

(a) 30° (b) 45° (c) 60° (d) 90°

Draw the following angles using compass.

(a) 30° (b) 60° (c) 120° (d) 90° (e) 45°

## 13.1 Construction of Angle by Using Compass

### Activity 1

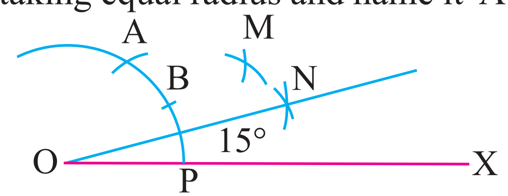
Construct an angle of 15° by using compass.

**Procedure:**

(a) Draw a line segment OX.

(b) Draw an arc by taking radius OP with center O.

(c) From the point P, cut the arc by taking equal radius and name it A where 60° angle is formed.




(d) From the point A and P take an arc and cut at common point and name it M.

(e) By using scale, from the point O and M, draw a dotted line and on arc AP, take a point B.

(f) From the point B and P, take an arc and cut each other at N.

(g) Now join N and O. Measure ∠NOP.

Hence ∠NOP = 15° is ready.

### Activity 2

Draw an angle of 75° by using compass.

**Procedure :**

(a) Draw a line segment AB.

(b) From the point A, draw an arc equal to AM. From the point M, cut the arc at C by taking equal radius. Again from the point C, cut at D where 60° and 120° is formed.

(c) From the point C and D, cut at common point N. Join A and N.

Name the point E where arc and line AN cuts.

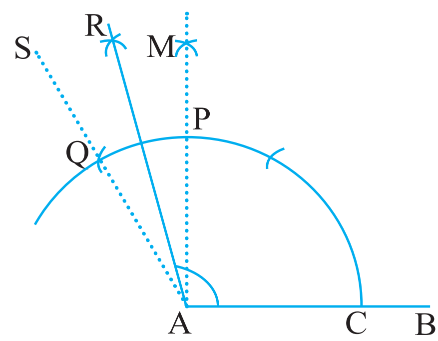
(d) From the point E and C, cut at common point K. Join A and K.

Now ∠KAB = 75° is formed.

## Activity 3

Construct an angle 105° by using compass.

**Procedure:**




(a) Draw a line segment AB.

(b) At point A, construct ∠SAB = 120° and ∠MAB = 90°

(c) From the point P and Q, cut at R by taking equal arc.

(d) Join R and A and measure ∠RAB.

Now ∠RAB = 105° is formed.

## Activity 4

Construct an angle 135° by using compass.

**Procedure:**

(a) Draw a line segment MAB.

(b) Taking center at A, draw an arc AC. Cut the arc by the same arc of AC. Mark this point as P. Again cut the arc by taking same arc AC and mark it as Q.

(c) From the point P and Q take some arc and cut at common point R. Now join R and A so that ∠RAC = ∠SAM = 900 is formed.

(d) From the point S and M, take some arc and cut at a common point. Write this name as K.

Now join K and A and measure ∠KAC.

Now ∠KAC = 135° is fuormed.

## Activity 5

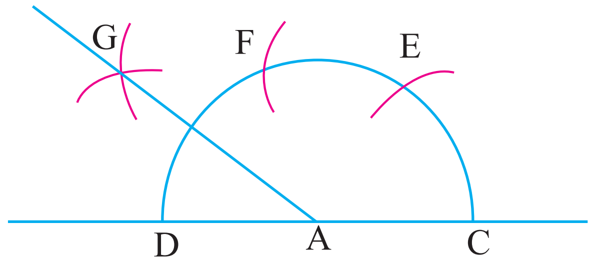
Construct an angle 150° by using compass.

**Procedure:**

(a) Draw a line segment DAC.

(b) From the point C, take some arc and cut that arc at E from C. From E again cut at F.

(c) Take some arc and cut at common point G from D and F.




(d) Now join G and A and measure it.

Now ∠GAC =150°

## Exercise 13.1

**1. Draw the following angles by using scale and compass.**

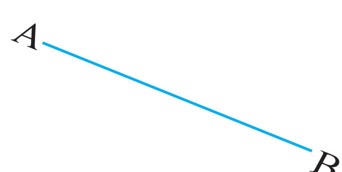
(a) 1 5° (b) 45° (c) 60° (d) 75°

(e) 105° (f) 135° (g) 150°

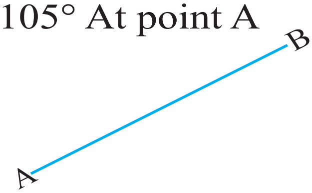
**2. Draw the following lines in copy and draw an angle at the given points.**

(a) 135° At point Q

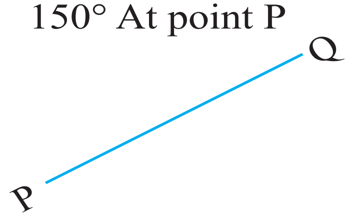

(b) 75° At point B




(c) 105° At point A




(d) 150° At point P



(e) 120° At point B


3. Draw an angle 150° and bisect it. Also measure the angle by protractor.

4. Draw a line segment AB and at A and B draw an angle 105° and 30°.

   Take this point C at intersection of the angles at A and B.

   Now measure the angle $\angle ACB$ by protractor.

5. Draw a line segment PQ = 6 cm and at P and Q draw an angle 135° and 150. Take this point R at intersection of the angles at P and Q.

   Now measure the angle $\angle PRQ$ by protractor.

**Answer**

Show the answers to your teacher.

## 13.2 Construction of Equal Angle Using Campass

### Activity 1

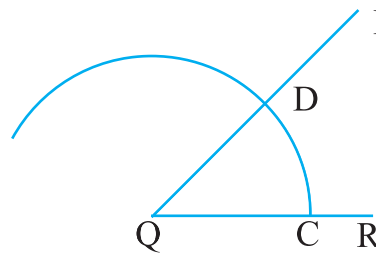
Draw an angle by a ruler. Construct another angle using a compas that is equal to the first one.

**Procedure:**

(a) Draw an angle ∠PQR by ruler.

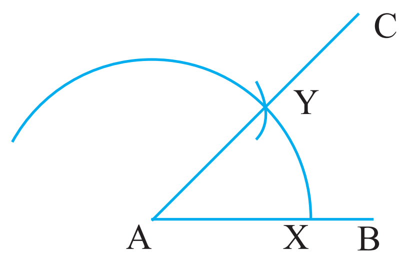
(b) Draw a line AB .

(c) At point Q take some arc equal to QC and cut at Cand D.




(d) Put the compass at A. Take the arc which is equal to previous one and draw an arc.

(e) Measure the length of C and D by compass and at Y from X which is shown in figure. Now join A and Y and extend it to C.



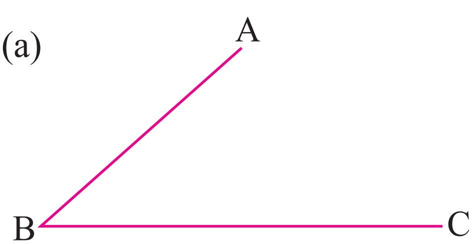

(f) Measure ∠PQR and ∠CAB by protractor and check whether it is equal to the previous one or not.

Hence, ∠PQR and ∠CAB are equal.

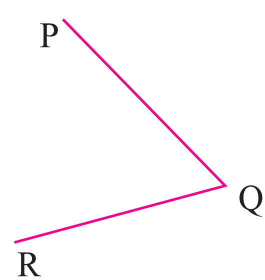
### Exercise 13.2

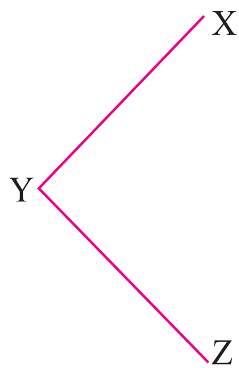
**1. Trace the following angles in your copy. Construct an angle equal to the following angle.**

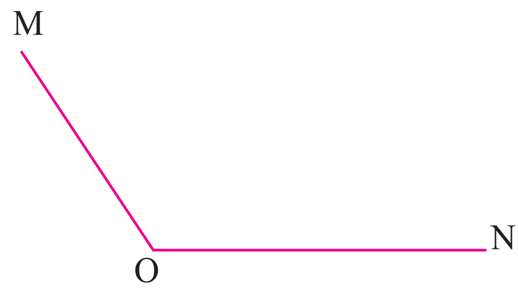
(a)




(b)



(c) 

(d) 





**2. Draw the following angles using Protractor and hence draw the equal angles by using compass.**

(a) 35° (b) 50° (c) 95° (d) 130° (e) 160°

**Answer**

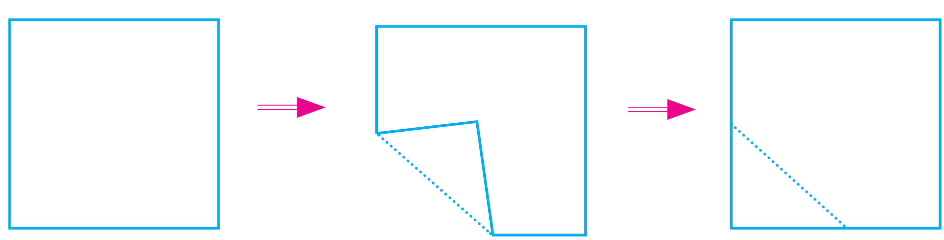
Show answers to your teacher.

## 13.3 Introduction to Pair of Angles

### 13.3.1 Adjacent Angles

**Activity 1**

Take a piece of paper. Fold the paper from a corner. Now open the folded paper. Draw straight lines in folded part and edges. Discuss about the following questions with friends.



(a) How many straight lines are there in the figure?

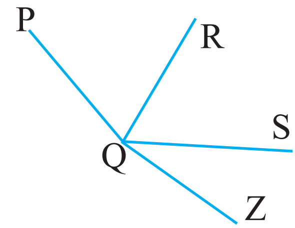
(b) How many angles and vertices are there in the figure?

(c) Is there any common side for different angles?

## Activity 2

From the point Q, draw lines QS, QR, QP and QZ.

What angles are formed while drawing the lines?



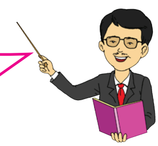

What is common in ∠PQR and ∠RQS, ∠RQS and ∠SQZ, ∠PQR and ∠RQZ? Discuss in class

Now, QR is common side in ∠PQR and ∠RQS.

QS is common in ∠RQS and ∠SQZ

QR is common in∠PQR and ∠RQZ.

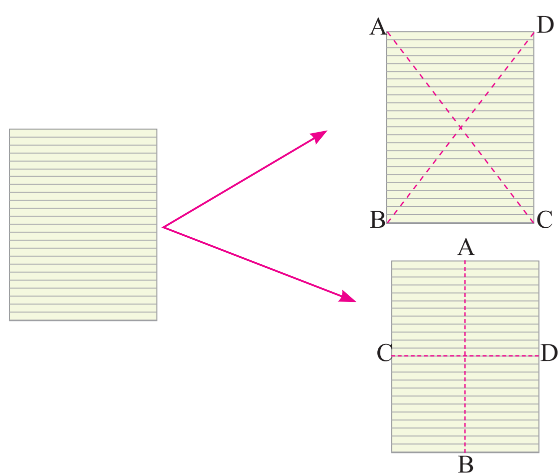
The angles having same vertices and angles on both side from the common side is called adjacent angle.



## 13.3.2 Vertically Opposite Angles

## Activity 3

Take a page of an exercise book. Fold the paper like in the given figure. Open the folded paper in which we can see the two lines intersected at a common point. Draw the lines in dotted part.




(a) How many angles are formed?

(b) Which angles are equal?

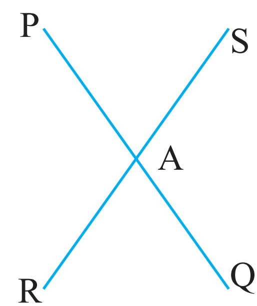
## Activity 4

Draw two intersecting lines PQ and RS and write the name A in intersected point.

Now measure the angles ∠PAS, ∠RAQ, ∠PAR and ∠QAS and discuss which angles are equal.

Here angles ∠PAS, ∠RAQ, ∠PAR and ∠QAS are in opposite direction and are equal.

Hence, angles ∠PAS and ∠RAQ or ∠PAR and ∠QAS are vertically opposite angles.

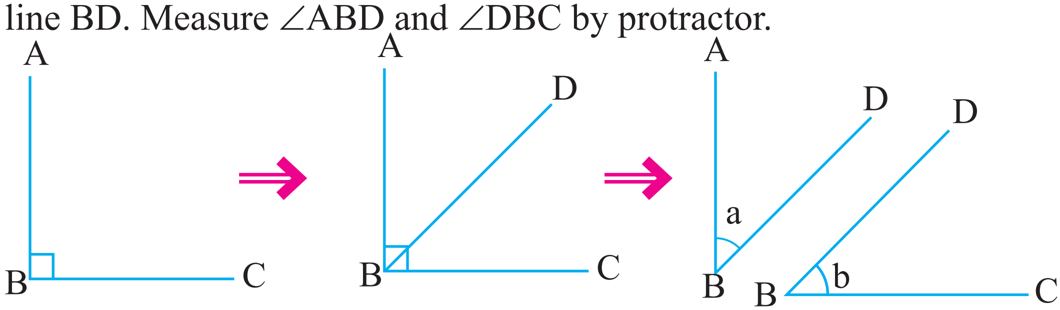



When two straight lines are intersected at a point, the angles in opposite sides are called vertically opposite angles.

## 13.3.3 Complementary Angles

## Activity 5

Draw an angle 90° by using set square. From the point B, draw another line BD. Measure ∠ABD and ∠DBC by protractor.



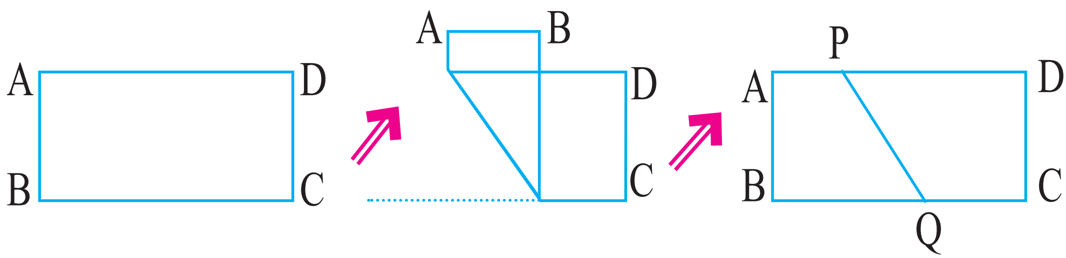

Now find the sum of ∠ABD and ∠DBC. The sum of these two angles is 90°.

Hence, if the sum of two angles is 90°, these two angles are called

## 13.3.4 Supplementary Angles

### Activity 6

Take a page of an exercise book. Write name A, B, C and D like in the figure. Fold the paper from the side of length. Write the name P and Q in the place where the paper is folded. Draw the line PQ.




Measure the angles ∠BQP + ∠PQC and ∠BQC and show in the class.

Now, ∠BQC =180°. So the sum of ∠BQC and ∠PQC is 180°

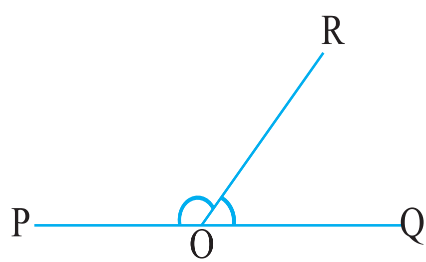
### Activity 7

Draw a straight line PQ and take any point O and draw OR line. Now measure ∠POR and ∠ROQ.

Find the sum of ∠POR and ∠ROQ.

Here ∠POR + ∠ROQ =180°

Hence ∠POR and ∠ROQ are supplementary angles.

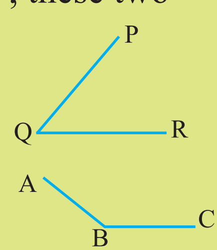



If the sum of two angles is two right angle or 180°, these two angles are called supplementary angles.

In the given figure ∠PQR = 70°, ∠ABC =110°

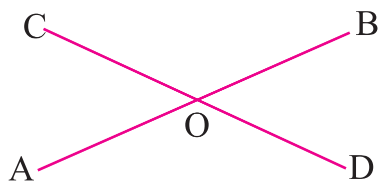
The sum of two angles is 180°

Therefore, ∠PQR is supplementary angle of ∠ABC.



## Example 1

**In the given figure the lines AB and CD are intersect at O. Then,**




(a) Write the vertically opposite angles of ∠AOD and ∠AOC

(b) Write the adjacent angles of ∠AOD

(c) What is the sum of ∠AOD and ∠BOC?

(d) Which angle is equal to ∠AOC? Write it.

(e) Write the supplementary angle of ∠BOC.

### Solution

Here,

(a) Here, the vertically opposite angle ∠AOD is ∠BOC and vertically opposite angle of ∠AOC is ∠BOD.

(b) The adjacent angle of ∠AOD are ∠AOC and ∠BOD.

(c) The sum of ∠BOD and ∠BOC is 180°

(d) The angles ∠AOC and ∠BOD are equal.

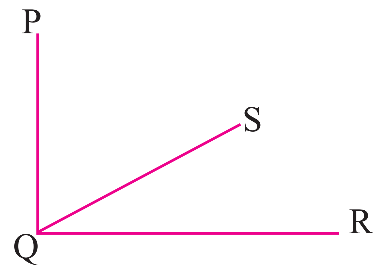
(e) The supplementary angle of ∠BOC are both ∠AOC and ∠BOD.

## Example 2

**In the given figure, lines PQ, QR and QS are drawn from Q where ∠POR = 90° then**

(a) Write adjacent angle of ∠PQS.

(b) Write complementary angle of ∠SQR.




**Solution**

Here, (a) The adjacent angle of ∠PQS is ∠SQR.

(b)The complementary angle of ∠SQR is ∠PQS.

## Example 3

**Find the complementary and supplementary angle of 33°**

**Solution**

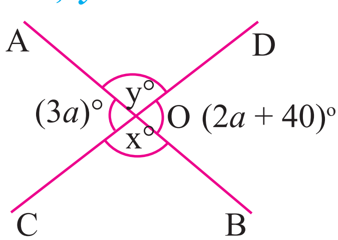
Given angle = 33°

The complementary angle of 33° = (90° – 33°) = 57°

The supplementary angle of 33° = (180° – 33°) = 147°

## Example 4

**Figure find the value of $x°$, $y°$ and $a°$ from the given fiture.**




**Solution**

Here,

(a) ∠AOC = ∠BOD [∵ vertically opposite angles]

or, 3a = 2a + 40°

or, a = 40°

(b) $\angle AOD + \angle AOC = 180°$ [∵ Straight angle]

or, $y + 3a = 180°$

or, $y + 3 \times 40 = 180°$ [∵ a = 40°]

or, $y = 180° - 120°$

or, $y = 60°$

(c) $\angle BOC = \angle AOD$

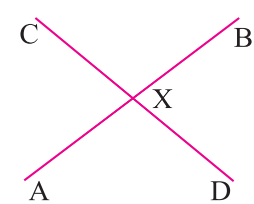
or, $x = y$

or, $x = 60°$

Therefore, $a = 40°$, $y = 60°$ and $x = 60°$

## Exercise 13.3

**1. From the given figure, write down the angles having the following relationship with ∠AXC.**




(a) Two adjacent angle

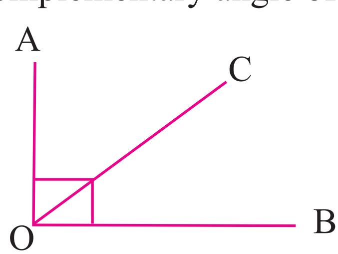
(b) Two supplementary angle

(c) One vertically angle

**2. In the given figure ∠AOB = 90°**

(a) Write adjacent angle of ∠AOC.

(b) Write the complementary angle of ∠BOC.

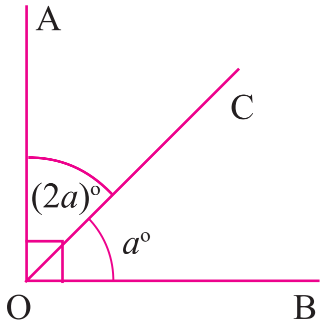



**3. Find the complementary and supplementary angles of the following angles.**

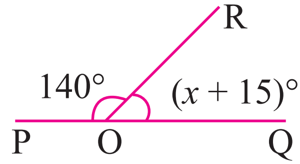
(a) 15° (b) 45° (c) 78° (d) 87°

**4. Find the value of x , y and a from the following figure.**

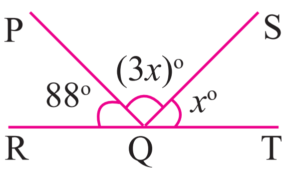
(a)



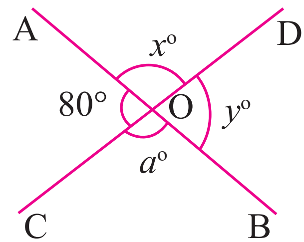

(b)



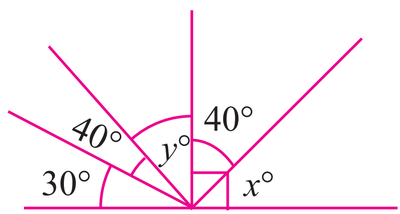

(c)




(d)




(e)




## Project Work

Discuss the possible pair of angles formed when two lines are intersected to each other present the findings in the class.

## Answer

1. (a) ∠BXC and ∠AXD (b) ∠AXD and ∠CXB (c) ∠BXD
2. (a) ∠BOC (b) ∠AOC
3. (a) 75°, 165° (b) 45°, 135°
   (c) 12°, 102° (d) 3°, 93°
4. (a) 30° (b) 25° (c) 23°
   (d) 100°, 80°, 100° (e) 50°, 20°

## 13.4 Experimental Verification of Angles

**Experiment 1**

When two lines are intersected to each other, the vertically opposite angles are equal.

Here,

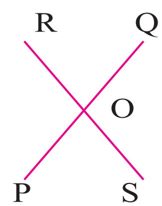



Figure 1

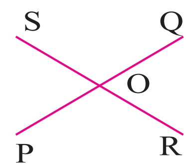



Figure 2

Draw two straight lines PQ and RS which intersect at O. Now, measure the angles ∠ROQ, QOS, ∠ROP and ∠POS by protractor and complete the following table.

| Figure | ∠ROP | ∠QOS | ∠ROQ | ∠POS | Result |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

**Conclusion:** When two lines are intersected to each other, the vertically opposite angles are equal.

**Experiment 2**

When the two lines are intersected the sum of adjacent angles on the same side is 180°.

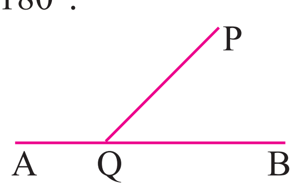



Figure 1

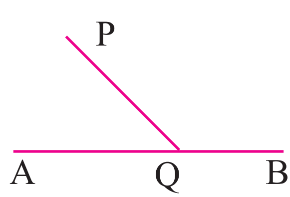



Figure 1

Construct two different figures and on the line AB take a point Q and draw a line QP. Measure the angles ∠PAQ and ∠PQB using the protractor and complete the following table.

| Figure | ∠PQA | ∠PQB | Result |
|---|---|---|---|
| 1 | | | |
| 2 | | | |

When the two lines are intersected the sum of adjacent angles on the same side is 180°.

## Experiment 3

The complete angle at a point is 360°.

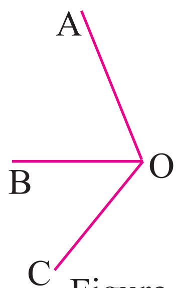



Figure 1

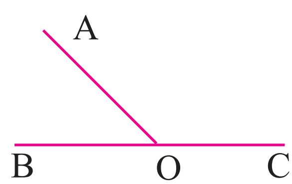


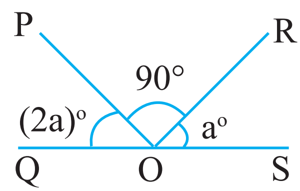

Figure 2

Draw two different figure as given above. Now measure the angles ∠AOB, ∠BOC and ∠AOC and fill in the following table.

| Figure | ∠AOB | ∠BOC | ∠AOC | ∠AOB + ∠BOC + ∠AOC | Result |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

**Conclusion:** The complete angle at a point is 360°.

## Example 1




**Find the value of a from the given figure.**

**Solution**

Here Here, ∠POQ + ∠POR + ∠ROS = 180° (The sum of angles in a straight line is 180°)

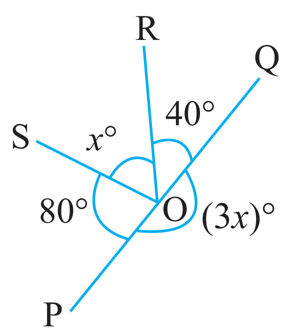
or, $2a + 90 + a = 180°$

or, $3a = 90°$

or, $a = 30°$

## Example 2

**Find the value of *x* from the given figure.**




**Solution**

The complete angle at a point is 360°.

Here ∠POQ + ∠ROQ + ∠ROS + ∠SOP = 360°

or, $3x + 40° + x + 80° = 360°$

or, $4x + 120° = 360°$

or, $4x = 360° – 120°$

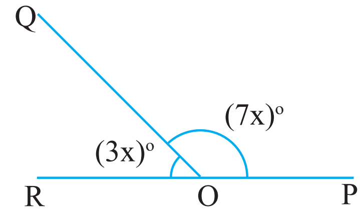
or, $4x = 240$

or, $x = \frac{240}{4}$

or, $x = 60°$

## Example 3

**From the given figure, find the value of $\angle POQ$ and $\angle QOR$.**




**Solution**

Here, $\angle POQ + \angle QOR = 180°$

or, $7x + 3x = 180$

or, $10x = 180$

or, $x = \frac{180}{10} = 18$

Now, $\angle POQ = 7x = 7 \times 18 = 126°$

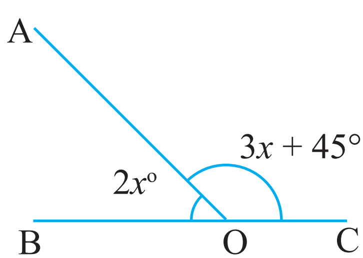
and $\angle QOR = 3x = 3 \times 18 = 54°$

Therefore, $\angle POQ = 126°$ and $\angle QOR = 54°$

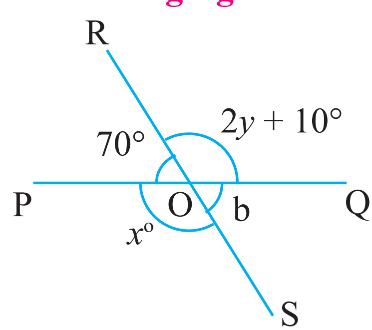
## Exercise 13.4

**1. Find the values of $x$, $y$ and $b$ from the following figure.**

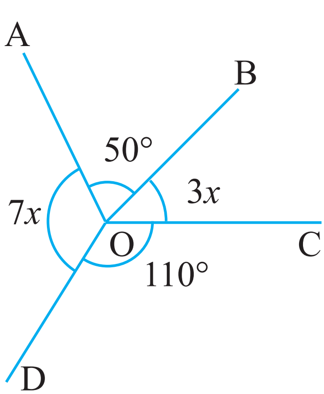
(a)



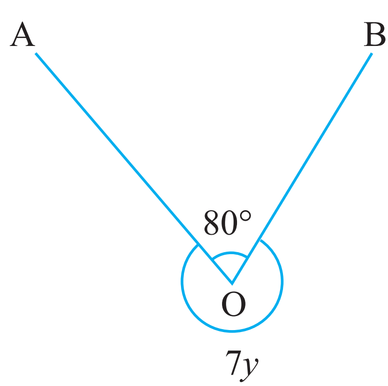

(b)




(c)




(d)



2. **Prove experimentally the following facts.**

(a) When two straight lines are intersected to each other, the vertically opposite are equal.

(b) The sum of adjacent angles on the same side is 180° at any point on a straight line.

(c) The complete angle at a point is 360°.

**Answer**

1. (a) 27° (b) $y = 50°$, $x = 110°$, $b = 70°$

   (c) $x = 20°$ (d) 40°

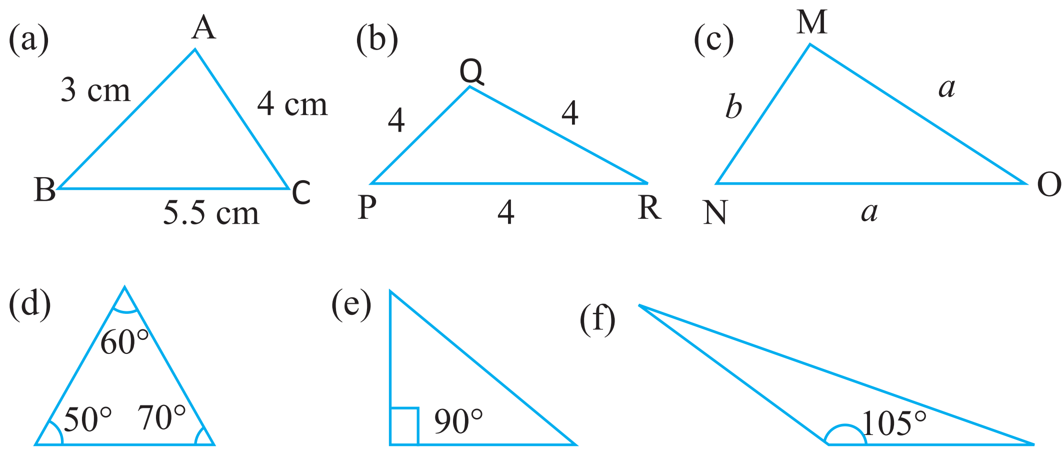
2. Discuss the solution of all the questions in the classroom.

Lesson 14

# Plane Figures

## 14.0 Review

State the types of triangles (equilateral, scalene, acute angled, right angled and obtuse angled) and discuss in the class.



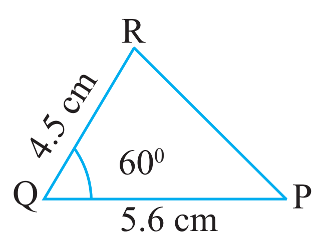

## 14.1 Construction of Triangle

### 14.1.1 Construction of triangle when two sides and angle between them is given.

**Activity 1**

Construct a triangle PQR where PQ = 5.6 cm, QR = 4.5 cm and ∠PQR = 60°

First draw a rough sketch.

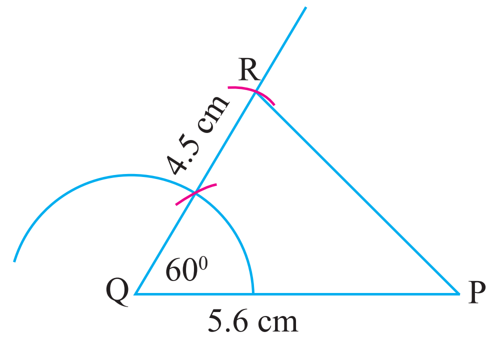



**Procedure:**

1. Draw a line PQ = 5.6 cm.
2. Draw 60° at point Q with the help of compass.

3. From the point Q cut QR = 4.5 cm.
4. Join P and R

Hence PQR is the required triangle.

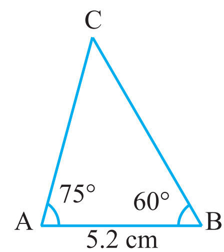



## 14.1.2 Construction of triangle when one side and two angles at the end are given.

### Activity 2

Construct a triangle ABC where AB = 5.2 cm, ∠A = 75° and ∠B = 60°

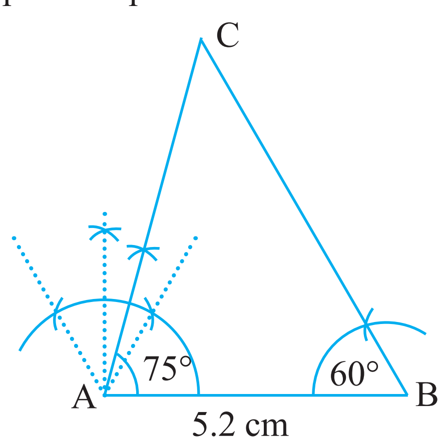
First draw a rough sketch.




### Procedure

1. Draw a line AB = 5.2 cm
2. Draw 75° angle at point A with the help of compass.
3. Draw 60° angle at point B with the help of compass.
4. Now draw two lines from A and B so that they meet at C.

Hence, ΔABC is the required triangle.



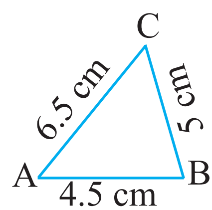
## 14.1.3 Construction of Triangle when the length of three sides are given.

### Activity 3

Draw a triangle ABC in which AB = 4.5 cm, BC = 5 cm and CA = 6.5 cm.

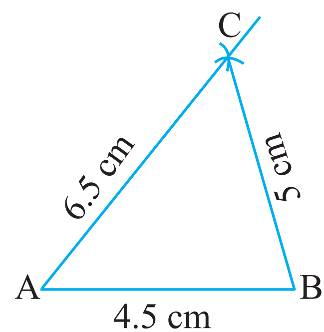
**Procedure**

First draw a rough sketch.




1. Draw a line segment AB =4.5 cm
2. From point A, take radius 6.5 cm and from point B, take radius 5 cm and cut at a common point C.
3. Join AC and BC.

Hence, ΔABC is the required triangle.

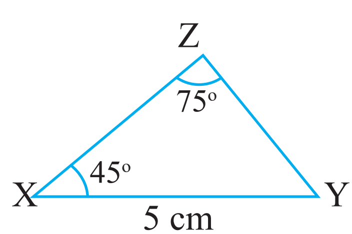



## 14.1.4 Construction of Triangle when one side, angle on that point and opposite angle of the side is given.

### Activity 4

Construct a triangle XYZ in which XY = 5 cm, ∠ZXY = 45° and ∠XZY = 75°

First draw a rough sketch.




**Procedure**

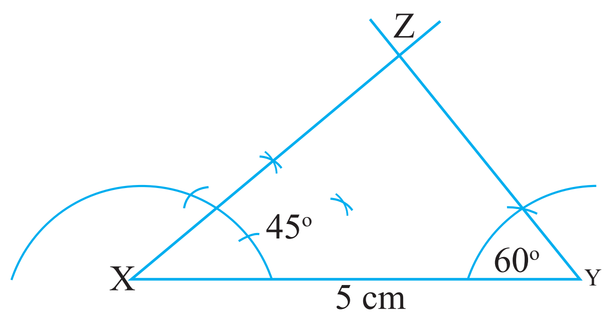
1. Draw a line segment XY = 5 cm

2. Construct 45° angle at the point X.

3. At point Y, construct [180° – (75° +45°) = 60°] i. e. 60° angle by compass.

4. Name the point Z where the lines from point X and Y meet each other.

Hence, ΔXYZ is the required triangle.




## Exercise 14.1

**1. Construct the triangle PQR under the following conditions.**

(a) PQ = 4.8 cm, QR = 5 cm and ∠PQR = 75°

(b) PR = 5 cm, ∠PRQ = 45° and QR = 5.8 cm

(c) PQ = 6.2 cm, ∠QPR = 60° and RQ = 6.6 cm

**2. Construct the triangle ABC under the following conditions.**

(a) ∠ABC = 60°, ∠ACB = 45° and BC = 6 cm

(b) AB = 6.8 cm, ∠BAC = 75° and ∠ABC = 30° cm

(c) CA = 5.2 cm, ∠ACB = 45° and ∠BAC = 75° cm

**3. Construct the triangle DEF under the following conditions.**

(a) DE = 4.5 cm, EF = 4 cm and DF = 5 cm

(b) EF = 6.6 cm, DF = 6 cm and DE = 7 cm

(c) DE = EF = 5.5 cm and DF = 5.2 cm

4. **Construct the triangle LMN under the following conditions.**

(a) LM = 6 cm, $\angle NLM = 60^{o}$ and $\angle LNM = 90^{o}$

(b) MN = 5.5 cm, $\angle LMN = 45^{o}$ and $\angle MLN = 60^{o}$

(c) LN = 7 cm, $\angle MLN = 60^{o}$ and $\angle LMN = 90^{o}$

5. **After discussing in an appropriate group, make questions to form the triangles by measuring the parts as given below. Then construct the triangles and present in the class.**

(a) Two sides and angle between them.

(b) Three sides given.

(c) One side and angles in the ends.

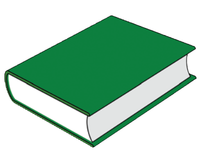
**Answer**

Show the answers to your teacher.

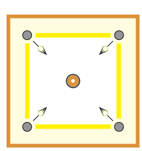
## 14.2 Identification and Verification of the Properties of Parallelogram, Rectangle and Square

**Activity 1**

Take a book, carromboard and a piece of cloth cut slanted. Discuss the faces/shapes of their surfaces.



book



carromboard

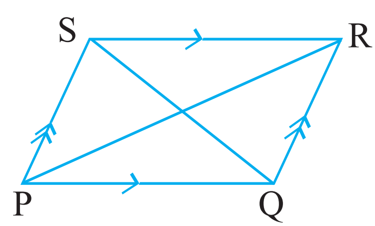
cloth

Now measure the angles formed in their corners and edges/sides. Discuss the characteristics of parallelogram on the basis of their angles and sides.

## 14.2.1 Identification of the Properties of Parallelogram

### Activity 2

Measure all the sides, angles and diagonals of the given parallelogram PQRS. Now discuss in the class the relationship between the sides, angles and diagonals of parallelogram. Find the characteristics of parallelogram.




### Experiment 1

Verify that the opposite angles of a parallelogram are equal.

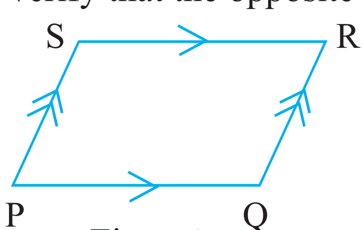
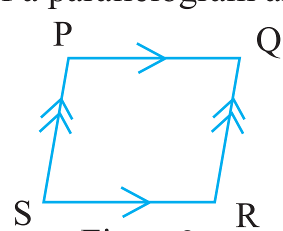


Figure1



Fiture 2

From the above parallelograms, measure all the angles and fill in the following table.

| Figure | ∠QPS | ∠PQR | ∠QRS | ∠RSP | Result |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

**Conclusion:** The opposite angles of a parallelogram are equal.

**Experiment 2**

Verify that the opposite sides of a parallelogram are equal.

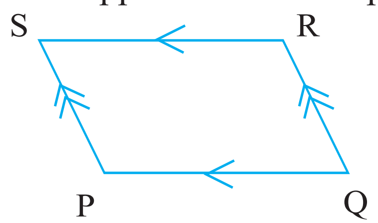



Figure 1

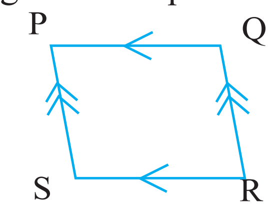



Figure 2

From the above parallelograms measure all the sides and complete the following table.

| Figure | PQ | QR | RS | SP | Result |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

**Conclusion:** The opposite sides of a parallelogram are equal.

**Experiment 3**

Verify that the diagonals of a parallelogram bisect to each other.

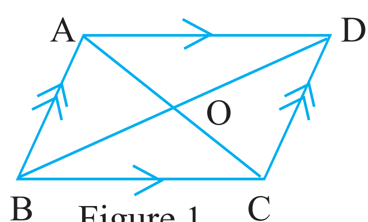



Figure 1

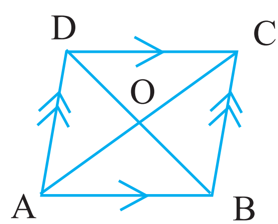



Figure 2

In both parallelogram, the diagonals AC and BD are intersected at O. Now measure the length of AO, OC, BO and OD and fill in the following table.

| Figure | AO | OC | BO | OD | Result |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

**Conclusion:** The diagonals of a parallelogram bisect to each other.

**Characteristics of a parallelogram:**

(a) The opposite angles of a parallelogram are equal.

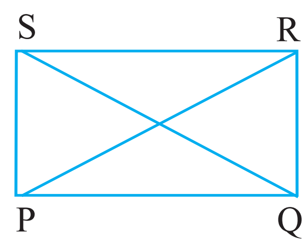
(b) The opposite sides of a parallelogram are equal.

(c) The diagonals of a parallelogram bisect to each other.

## 14.2.2 Identification of the Properties of Rectangle

### Activity 3

Take a piece of paper. Measure all the sides, angles and diagonals of it. What conclusion can you draw? Discuss in class.




### Experiment 1

All the angles of a rectangle are 90°.

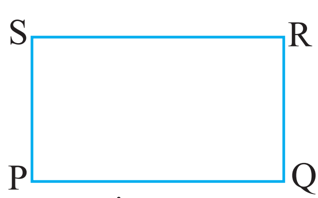


Figure 1

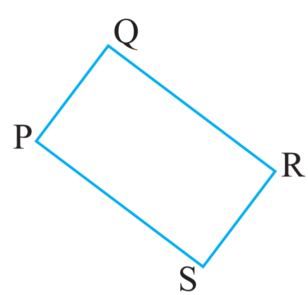


Figure 2

From the above two rectangles, measure all the angles and fill in the following table.

| Figure | ∠QPS | ∠PQR | ∠QRS | ∠RSP | Result |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

**Conclusion:** All the angles of a rectangle are 90°.

## Experiment 2

Opposite sides of a rectangle are equal.

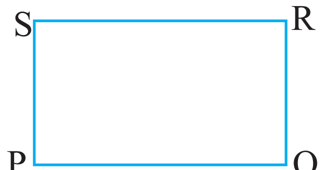


Figure 1

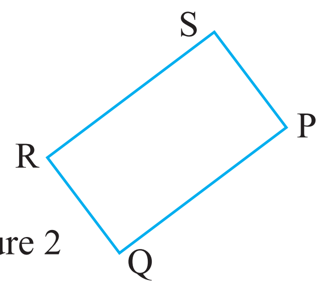


Figure 2

Measure all the sides of the above rectangle PQRS and fill in the following table.

| Figure | PQ | QR | RS | SP | Result |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

Conclusion:
Opposite sides of a rectangle are equal.

## Experiment 3

The diagonals of a rectangle are equal.

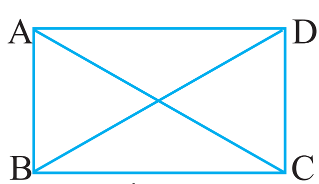


Figure 1

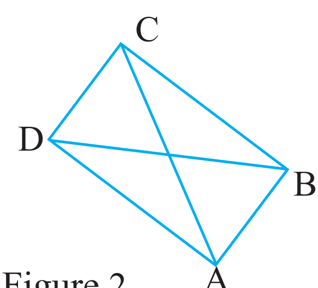


Figure 2

From the above rectangles, measure the two diagonals AC and BD and fill in the following table.

| Figure | AC | BD | Result |
|---|---|---|---|
| 1 | | | |
| 2 | | | |

**Conclusion:** The diagonals of a rectangle are equal

Characteristics of a rectangle:

(a) All the angles of a rectangle are 90°.

(b) The opposite sides of a rectangle are equal.

(c) The diagonals of rectangle are equal.

## 14.2.3 Identification of the properties of square

### Activity 4

Take a small chess board or a square faced solid object. Put the chess board or solid object on your copy and mark around it. Draw the diagonals. Now measure all the sides, angles, length of diagonals, angle between the diagonals and angle made by diagonals with side. What conclusion can you draw ? Discuss in class.

### Experiment 1

All the angles and sides of a square are equal.

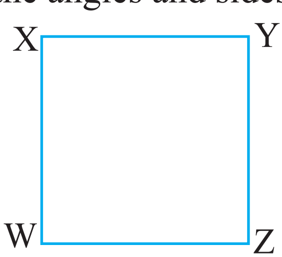


Figure 1

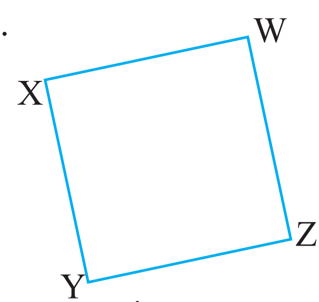


Figure 2

Measure all the sides and angles of the square WXYZ and fill in the following table.

| Figure | ∠X | ∠Y | ∠Z | ∠W | XY | WX | YZ | ZW | Result |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |

**Conclusion:** All the angles and sides of a square are equal.

## Experiment 2

The diagonals of a square are equal.

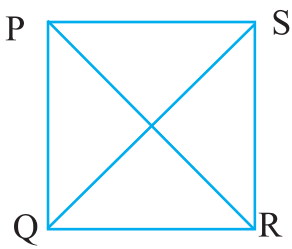



Figure 1

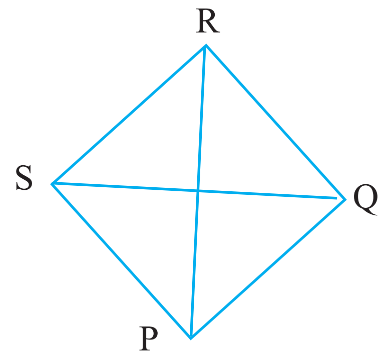



Figure 2

Measure the length of diagonals PR and QS of square PQRS and fill in the following table.

| Figure | PR | QS | Result |
|---|---|---|---|
| 1 | | | |
| 2 | | | |

**Conclusion:** The diagonals of a square are equal.

## Experiment 3

**The diagonals of a square bisect perpendicularly.**

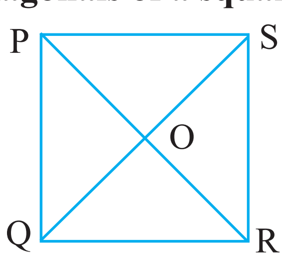



Figure 1

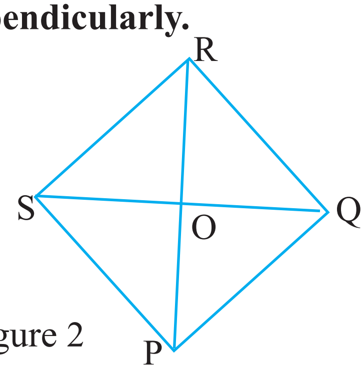



Figure 2

Measure the length of parts of diagonals and angle between diagonals and fill in the following table.

| Figure | PO | OR | QO | OS | ∠POQ | ∠POS | Result |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | | | | | | |

**Conclusion:** The diagonals of a square bisect perpendicularly.

## Experiment 4

**The diagonal bisect vertex angles in a square.**

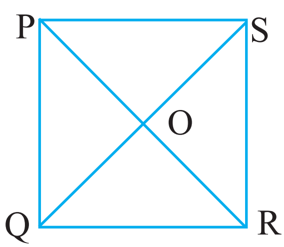



Figure 1

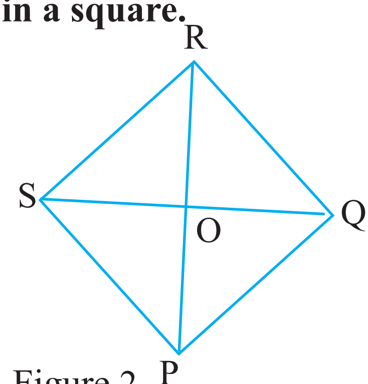



Figure 2

Measure the vertex angle and angles between diagonal and sides of a square and fill in the following table.

| **Figure 1** | | | |
|---|---|---|---|
| **Angle of vertices** | **Measure of subsidiary angle** | | **Result** |
| ∠PQR = ......... | ∠PQS = ......... | ∠SQR = .......... | |
| ∠QRS = ......... | ∠QRP = .......... | ∠PRS = .......... | |
| ∠RSP = ............ | ∠RSQ = .......... | ∠QSP = .......... | |
| ∠SPQ = ........... | ∠SPR = .......... | ∠RPQ = .......... | |

| **Figure 2** | | | |
|---|---|---|---|
| **Angle of vertices** | **Measure of subsidiary angle** | | **Result** |
| ∠PQR = ........... | ∠PQS = .......... | ∠SQR = .......... | |
| ∠QRS = ........... | ∠QRP = .......... | ∠PRS = .......... | |
| ∠RSP = ............ | ∠RSQ = .......... | ∠QSP = .......... | |
| ∠SPQ = ............ | ∠SPR = .......... | ∠RPQ = .......... | |

**Conclusion:** The diagonal bisect vertex angles in a square.

**Characteristics of a square:**

(a) All the angles and sides of a square are equal.

(b) The diagonals of a square are equal.

(c) The diagonals of a square bisect perpendicularly.

(d) The diagonal bisect vertex angles in a square.

## Exercise 14.2

**State whether the following statements are true or false.**

(a) The opposite sides of a parallelogram are equal.

(b) The opposite angles of a parallelogram are equal.

(c) Only the opposite sides of a square are equal.

(d) The diagonals of a rectangle bisect perpendicularly.

(e) All the characteristics of rectangles are also in parallelogram.

(f) The diagonals of a square are equal.

(g) Only opposite angles of a rectangle are equal.

## Project Work

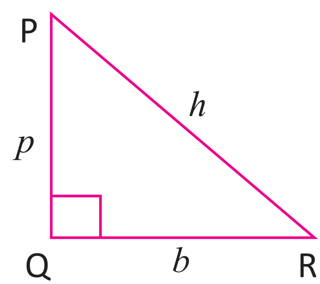
Collect the different objects having the shapes like square, rectangle, parallelogram from your school or home. Trace the objects in your exercise book. Test the characteristics of rectangle, square and parallelogram and present in the class.

## Answer

Answer: Show the answers to your teacher.

## 14.3 Pythagoras Theorem

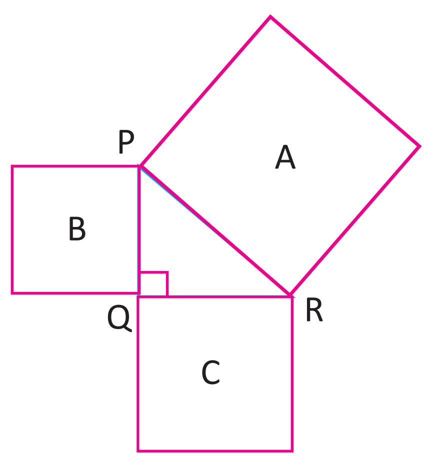
### Experiment of Pythagoras theorem




Draw a right angled triangle in which $\angle PQR = 90°$. Then hypotenuse = $h$, perpendicular = $p$ and base = $b$.

How can we find hypotenuse, perpendicular and base in a right angled triangle?

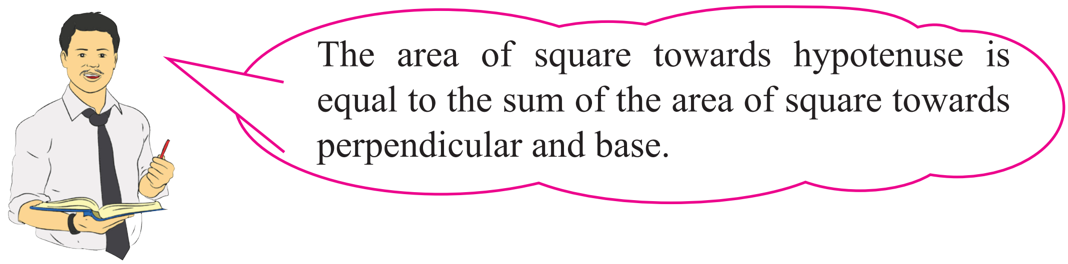
Measure all the sides of the triangle and make square in each side of it.




Now, find the area of square A,B and C. Discuss in class. Is the area of square towards hypotenuse equal to the sum of the area of square towards perpendicular and base?

Here, the area of square towards hypotenuse is equal to the sum of the area of square towards perpendicular and base.

Hence $h^2 = p^2 + b^2$

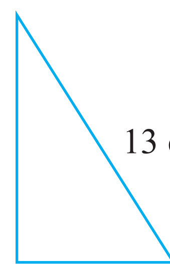



## Activity 1

Take a setsquare. Measure all the side of it. Find the longest side of it. Discuss whether the area towards longest side is equal to the sum of the squares towards perpendicular and base or not. What conclusion can you draw?

## Example 1

Test whether this triangle is right angled or not?




**Solution**

*Here,*

$h = 13$ cm

$p = 12$ cm

$b = 5$ cm

Now, $h^2 = p^2 + b^2$

or, $(13)^2 = (12)^2 + (5)^2$

or, $169\ \text{cm}^2 = 144\ \text{cm}^2 + 25\ \text{cm}^2$

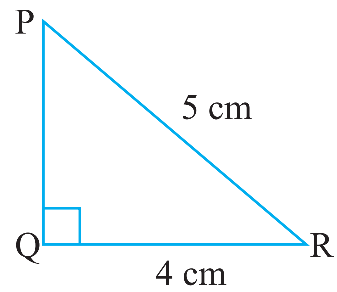
or, $169\ \text{cm}^2 = 169\ \text{cm}^2$

Here, area of square towards hypotenuse is equal to the sum of area of squares towards perpendicular and base. Hence, it is a right angle triangle.

## Example 2

Find the unknown side from the given right angled triangle.

**Solution**




Here, $\angle PQR = 90°$

$h = PR = 5$ cm

$b = QR = 4$ cm

$p = PQ = ?$

Now, $h^2 = p^2 + b^2$

or, $5^2 = (PQ)^2 + 4^2$

or, $25 = PQ^2 + 16$

or, $25 - 16 = PQ^2$

or, $PQ^2 = 9$

or, $PQ = 3$ cm

Therefore, the length of PQ is 3 cm.

## Exercise 14.3

**1. Write 'T' for the true and 'F' for the false sentences.**

(a) The longest side in any triangle is hypotenuse.

(b) The Pythagoras theorem is satisfied only in right angled triangle.

(c) In right angled triangle, the side which makes right angle is hypotenuse.

(d) The opposite side of right angle in right angled triangle is hypotenuse

**2. Which of the following triangles with the given measurement are right angled?**

(a) 12 cm, 10 cm and 5 cm

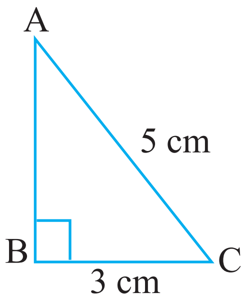
(b) 13 cm, 12 cm and 5 cm

(c) 15 cm, 16 cm and 17 cm

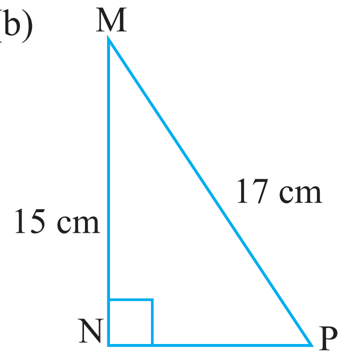
(d) 8 cm, 15 cm and 17 cm

3. **Find the unknown side of the following triangles.**

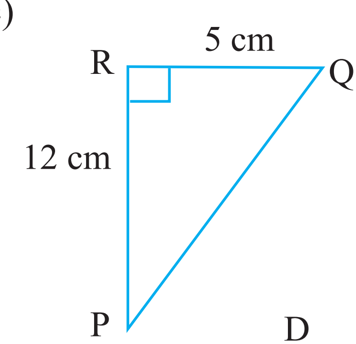
(a)



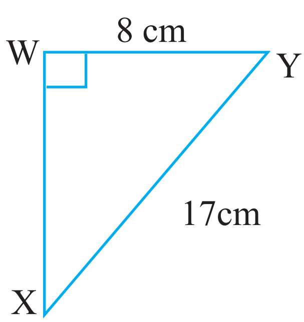

(b)



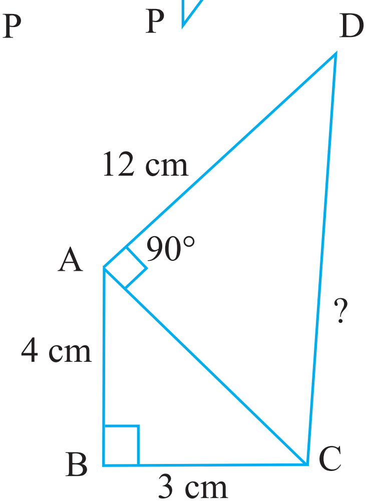

(c)




(d)




(e)




## Project Work

Collect triangular objects found your home and school. Measure the length and test whether Pythagoras theorem is satisfied or not. Show your work in class.

## Answer

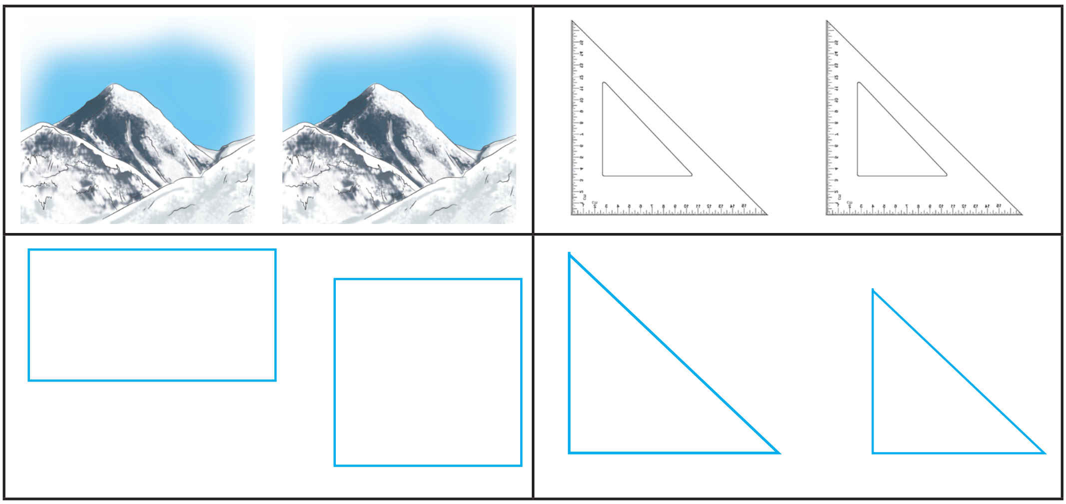
| | | | | |
|---|---|---|---|---|
| 1. (a) F | (b) T | (c) F | (d) T | |
| 2. (a) No | (b) yes | (c) no | (d) yes | |
| 3. (a) 4 cm | (b) 8 cm | (c) 13 cm | (d) 15 cm | (e) 13 cm |

Lesson 15

# Congruent Figures

## 15.0 Review

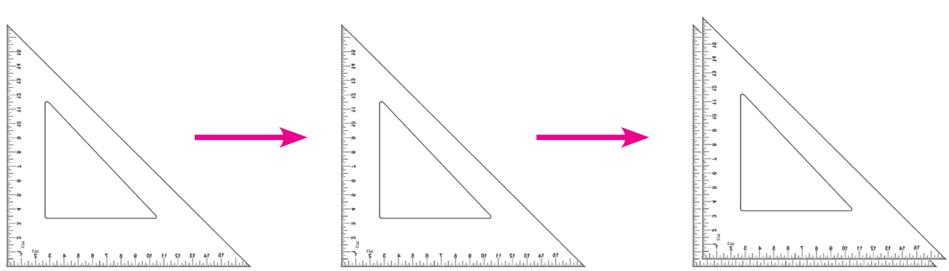
Which of the following figures have same shape and are equal in size?



## 15.1 Congruent Figures

**Activity 1**

Divide all the students of your class in four groups, Let the students take their own set square and keep in overlapped form. Compare and find the answers of the following questions.



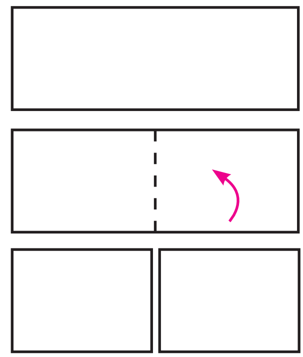
Overlapped set square

(a) Which shape are set square?

(b) Have the overlapped set squares same measurement or not?

(c) Demonstrate the set square having same shape and size in the class room. Are these set squares congruent ? Discuss in class.

## Activity 2

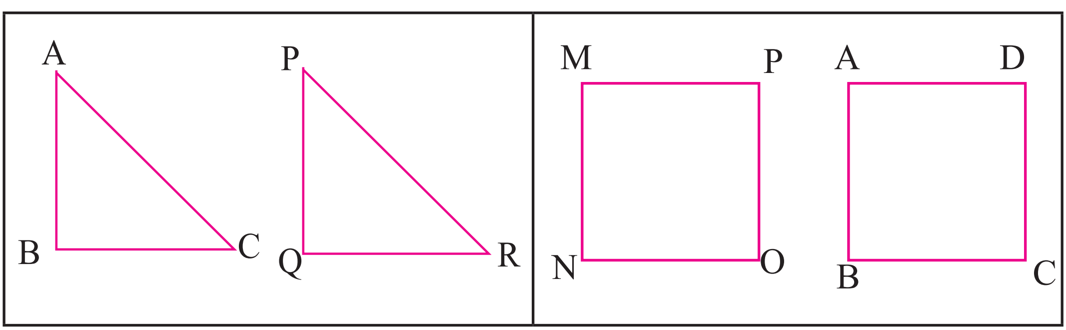


Take one one rectangular piece of paper. Fold it from the middle as shown in the figure. Cut the paper from the middle part. Put in overlapping form. Now compare both the pieces and find the answers of the following questions.

(a) Are both the pieces in same shape?
(b) Are the length of both shapes same?
(c) If both the objects have same shape and size then what kind of objects are they?

## Activity 3

Make groups of friends for each bench and draw the following pairs of figures with the help of tracing paper.




Now cut the papers from sides by scissors. Keep in overlapping form. Answer the following question now.

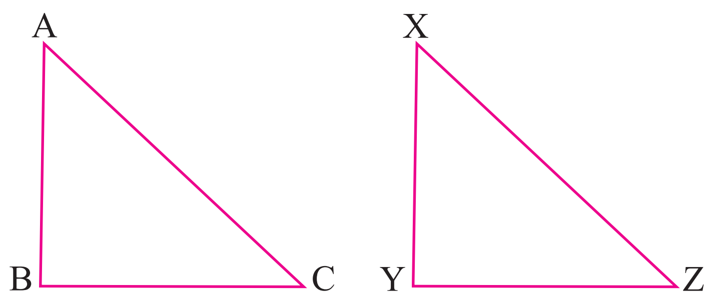
(a) Are the first pairs (triangles) of same shape?

(b) Are the first pairs (triangles) equal in size?

(c) What kind of similarities are there in second pairs of figures? Discuss in class and present its conclusion.

## Activity 4

Take one set square. Put the set square in your exercise book and trace two triangle from its outer border. Name these triangles as ΔABC and ΔXYZ.




Put these triangles in overlapping form and complete the table below.

| |
|---|
| In triangle XYZ, at point X there is a point ... of triangle ABC.<br>Over the point Y there is ...<br>Over the point Z there is …<br>Similarly,<br>Over the side XY there is …<br>Over the side YZ there is …<br>Over the side ZX there is …<br>So, XY = .........., YZ = ............., ZX = ................. . |

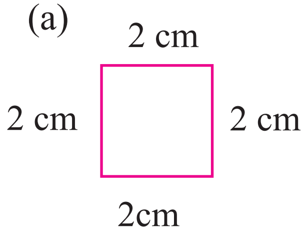
What kind of triangles are ABC and XYZ? Discuss with your friend and present in the class.

Conclusion: The figures having same shape and size are called congruent figures.

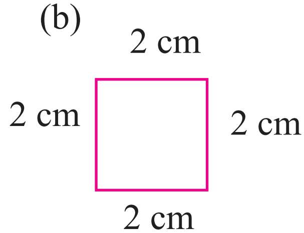
## Example 1

**Which of the following figures are congruent and why?**

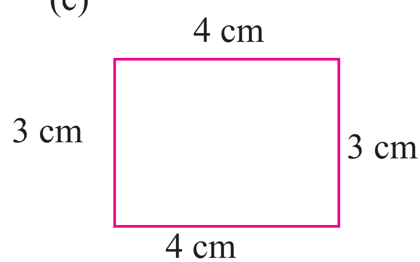
(a)




(b)




(c)




**Solution**

Here, figures (a) and ( b) are congruent because they have same shape and size.

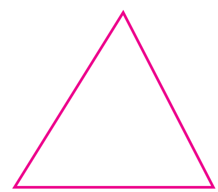
## Example 2

Which of the following figures are congruent and why?

(a)

(b)

(c)



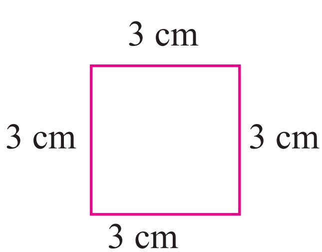
**Solution**

Here, figures (a) and (b) are congruent because they both have same shape and their sides are equal.

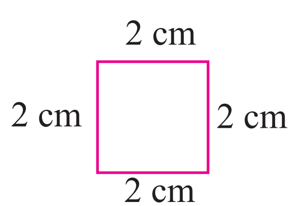
## Exercise 15

**1. Which of the following figures are congruent? And why?**

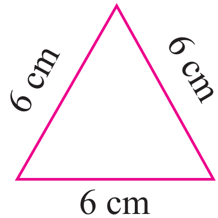
(a) (i)



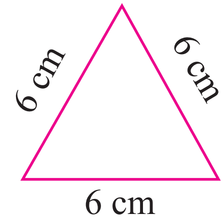

(ii)



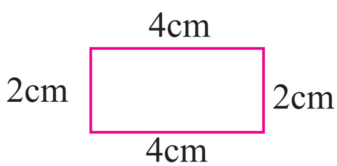
(b) (i)



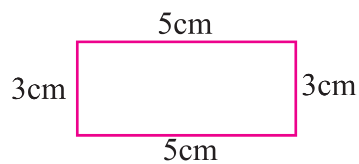

(ii)




(c) (i)

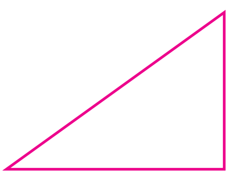



(ii)

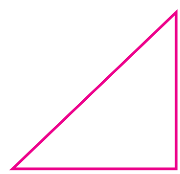



**2. Which of the following figures are congruent? Find it out by measuring the length.**

(a) (i)

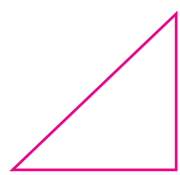


(ii)

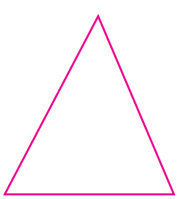


(b) (i)

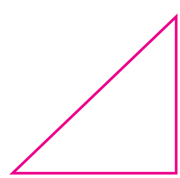
(ii)



(c) (i)

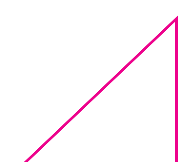


(ii)

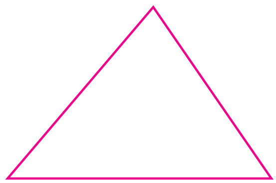


(d) (i)

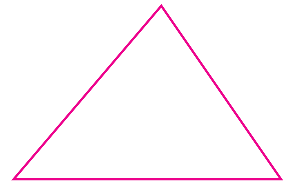
(ii)



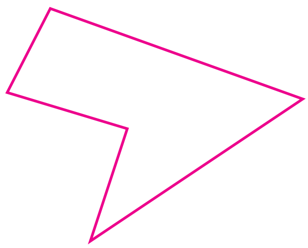
(e) (i)



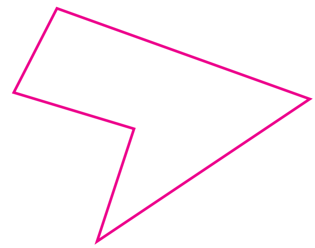
(ii)



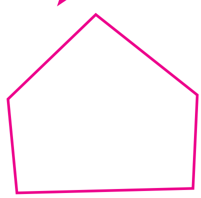
(f) (i)



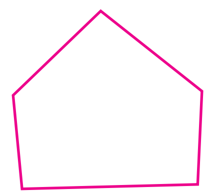
(ii)



(g) (i)



(ii)



3. Take five–five solid objects and draw congruent figures by using it.
4. Do "namaskar" by joining your hands. Are both the hands congruent to each other? Discuss in class.
5. Take two different coins of Rs.1 and Rs.2. Are these coins congruent to each other ? Discuss with your family members.

## Project Work

Divide all the students in different groups. Collect the congruent objects like coin, handkerchief, book, eraser etc that are found around the house or the school. Demonstrate them in the class.

## Answers

Show the answers to your teacher.

# Lesson 16 Solid Objects

## 16.0 Review

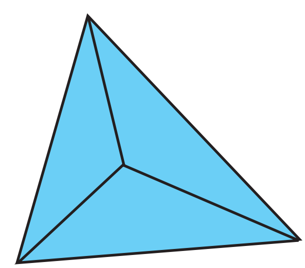
Discuss with your friend and fill in the following table.

| Name of objects | Plane figures (surfaces) | Solid objects |
|---|---|---|
| Box of matches | Rectangle | Cuboid |
| Dice | Square | Cube |
| Cone of ice cream | Circle | |
| Drum | | |

Discuss the solid objects and plane surfaces in the table above and present the conclusion in the class.

## 16.1 Tetrahedron

**Activity 1**



Divide the students into appropriate group. Provide the objects as shown in the figure to each group of students. Study about the object and discuss the following questions.

(a) Are all the edges equal?

(b) Are all the faces in equilateral triangular form?

(c) How many faces are there?

(d) How many edges and vertices are there?

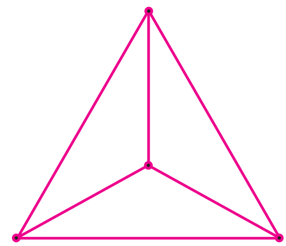
(e) What type of object is this ? Regular or irregular?

Present the conclusion in class.

Tetrahedron is a regular geometrical solid object. Its all faces are formed from equilateral triangle. There are 4 faces, 4 vertices and 6 edges in tetrahedron.

## 16.1.1 Skeleton of Tetrahedron

**Activity 2**



Divide the students in a certain group. Each group will take 6 small pieces of equal pipes (juice pipe) and 4 pieces of potato or other soft things. Now join the pipes and pieces of potato like in figure.

Now study the object and answer the following.

(a) What is the name of this object?

(b) How many edges and vertices are there?

Present the conclusion in class.

## 16.2 Octahedron

**Activity 3**

Take the object (as shown in the figure) in each group. Study and discuss about the following questions.

(a) Are all the edges equal?

(b) Are all the plane figures in equilateral triangular form?

(c) How many faces are there?

(d) How many edges and vertices are there?

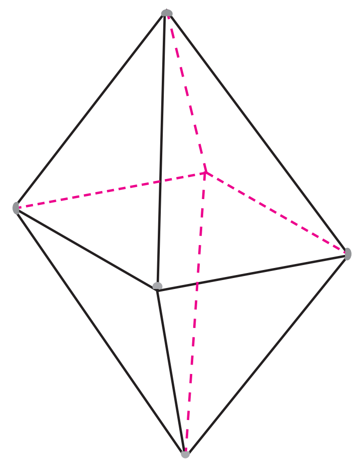
(e) What is the name of this solid objects?

(f) What type object is this ? Regular or irregular?

Octahedron is a regular solid object. Its all faces are in equilateral triangular form. It has 8 faces, 6 vertices and 12 edges.

## 16.2.1 Skeleton of Octahedron

### Activity 4



Divide the students in a certain group. Each group take 12 small pieces of equal pipes (juice pipe) and 6 pieces of potato or other soft things. Now join the pipes and pieces of potato like in the figure.

Now study the object and answer the following.

(a) What is the name of this object?

(b) How many edges and vertices are there?

(c) How many faces are there?

Present the conclusion in class.

### Activity 5

Divide all the students into five groups. Each group will collect and observe the solid model of cube, tetrahedron, octahedron, dodecahedron and icosahedron respectively. Now discuss the following questions.

(a) How many edges are there in these solid objects?

(b) How many plane surfaces are there in these solid objects?

(c) How many vertices are there in these solid objects?

Now fill in all the number of edges, plane faces and vertices in the following table. Discuss their relationship and present in the class.

| S.N. | Solid objects | No. of vertices(V) | No. of edges(E) | No.of faces (F) | Relation between V, E and F |
|---|---|---|---|---|---|
| 1. | Cube | | | | |
| 2. | Tetrahedron | | | | |
| 3. | Octahedron | | | | |
| 4. | Dodecahedron | | | | |
| 5. | Icosahedron | | | | |
| | | | | | |

From the above table, the relations between V, E and F is
V – E + F = 2.

**Example 1**

If a tetrahedron has 4 vertices and 6 edges, then find number of faces.

**Solution**

Here no.of vertices (V) = 4
No. of edges (E) = 6
No. of faces (F) = ?

We know that,

$V - E + F = 2$

or, $4 - 6 + F = 2$

or, $-2 + F = 2$

or, $F = 2 + 2$

$F = 4$

## Exercise 16.1

**1. Distinguish whether the following statements are true or false.**

(a) All the faces of tetrahedron are in equilateral form.

(b) All the edges of tetrahedron are equal.

(c) There are only three faces in tetrahedron.

(d) All the faces of octahedron are not in equilateral triangular form.

(e) There are only 4 edges in octahedron.

**2. Answer the following questions.**

(a) What is tetrahedron?

(b) Write any two differences between tetrahedron and octahedron.

(c) Write the formula to find the relation among the faces, edges and vertices of dodecahedron and icosahedron.

(d) What is the shape of faces in octahedron?

(e) How many vertices and edges are there in octahedron?

3. The number of edges and faces in a tetrahedron are 6 and 4 respectively. Find the number of vertices.

4. How many faces are there in a octahedron if it has 6 vertices?

### Project Work

Collect pipe of juice, straw, bamboo stick, cover of dotpen and rope. Then make the skeleton of cube, tetrahedron and octahedron using these objects and demonstrate in the class.

### Answer

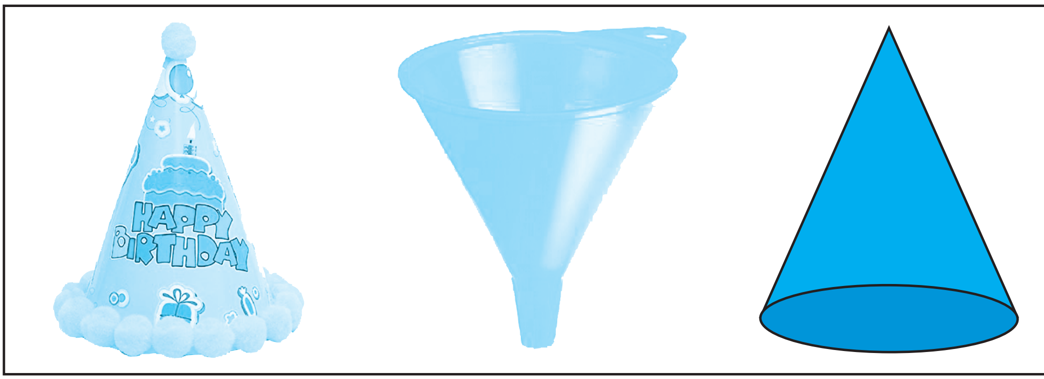
Show the answers to your teacher.

## 16.3 Cone and Cylinder

### 16.3.1 Cone

**Activity 1**

Sit in appropriate groups. Take one object as shown in the figure for each group. Observe the objects, discuss in the group and answer the following questions.




(a) What type of object are they?

(b) How is the base of each figure?

(c) What kind of faces are there in these object?

(d) How many vertices and edges are there in these objects?

In each above objects there are one vertex, one circular base and one circular face. These all solid objects are cone.

A solid object having one vertex, one circular base and one curved surface is called cone.

**Characteristics of cone**

(a) It has one vertex.

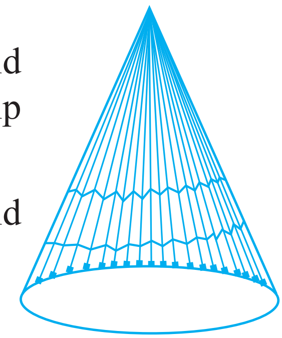
(b) It has one circular base.

(c) It has curved surface.

## Skeleton Model of Cone

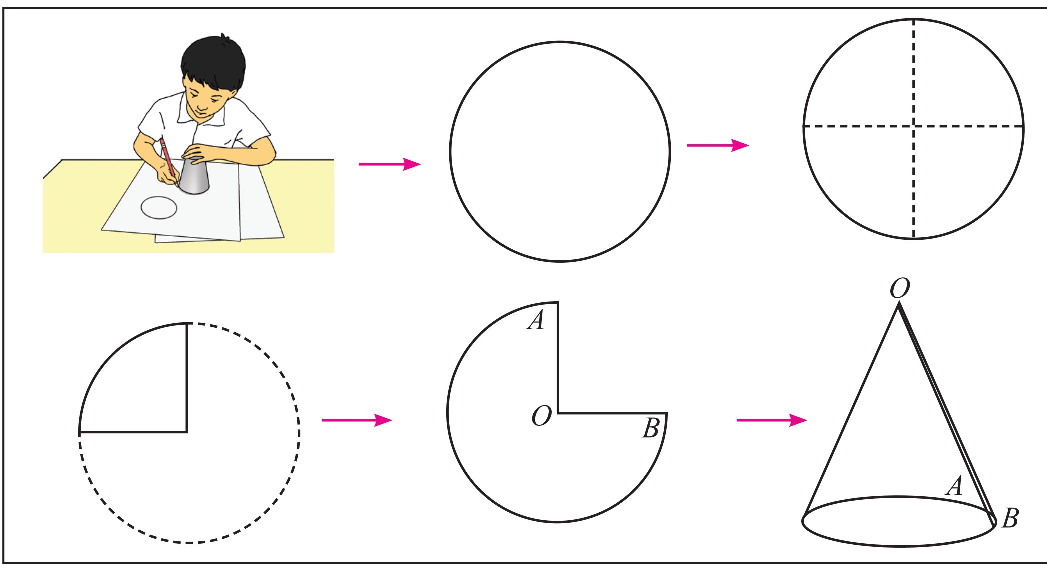
### Activity 2

Collect the samples like cone of ice cream, a tool which is used to kill the fish (Dhadiya) and discuss in the class.



Work in small groups in the class. Take circular solid object and a piece of paper. Make a circle by the help of the base of cone.

Cut the outer part with the help of scissors.Then fold the paper from the middle like in the figure.



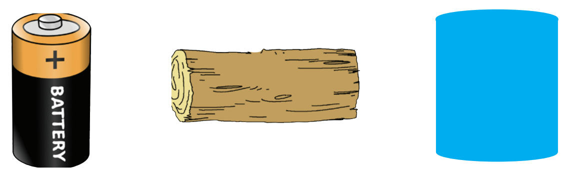

Now remove the one-fourth part of the circle by scissors. Paste the remaining part as shown in the figure. What kind of object did you get?

Now observe and find the number of vertices, corners and number of circular faces. Present it in the class.

## Cylinder

### Activity 3

Work in small groups in the class. Take the solid objects like in the figure. Study about the objects and answer the following questions after discussion in the group.




(a) What type of objects are they?

(b) How is the base of these objects?

(c) Is it possible to roll each objects?

(d) How many plane surfaces are there? count it.

There are two circular faces in all the solid objects.These objects are in cylindrical form.

### Characteristics of Cylinder

(a) Its bases are in circular form.

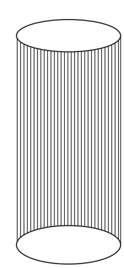
(b) It has one circular face.

(c) Its bases are parallel to each other.

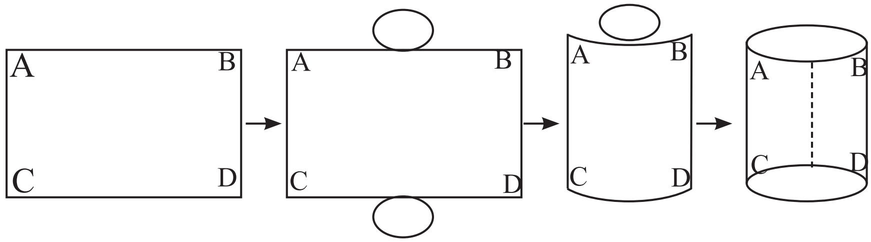
A cylinder is a solid object which has two circular parallel faces and one curved surface.

## 16.3.2 Skeleton Model of Cylinder

### Activity 4



List out the model of skeleton of cylinder that you have seen around your house, school and your localities. Take one piece of rectangular card board paper by each group. Take two circle whose circumference should be equal to the length of rectangular paper.




Roll the rectangular paper like in the figure and paste the circle in both upper and lower part.

What shape is formed? Write its name. How many circular faces and vertices are there? Observe them, discuss in the group and present it in the class.

## Exercise 16.2

1. **Answer the following questions.**
   (a) What is cone? Write any two merits of it.
   (b) What is cylinder? Write any two merits of it.
   (c) Write one similarity and one difference between cone and cylinder.
   (d) Make a sample figure of cone and cylinder.
2. What is the shape of base and surface of cylinder? Write.
3. Collect any five cylindrical objects. Write about their curved surface and bases.

## Project Work

Collect any five cylindrical and conical objects that are in your house and demonstrate them in the class.

## Answer

Show the answers to your teacher.

# Lesson 17 Coordinates

## 17.0 Review

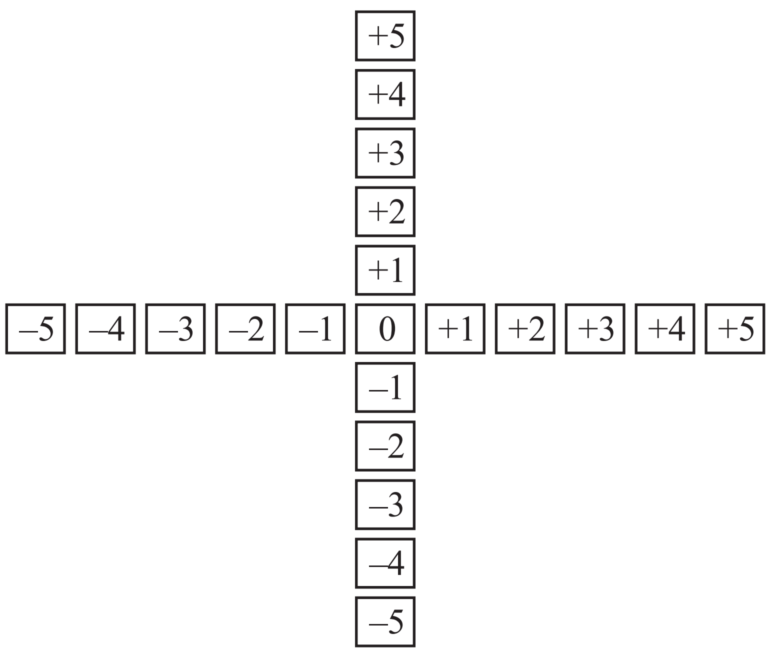
Together with your friends, make a number line along the X-axis and the Y-axis on pieces of paper as shown in the picture.

Now, be divided into two groups A and B and play the coordinate game.

### Rules for the game

First of all keep all the number cards in the perpendicular form in the playground as shown in the figure.

(a) Be divided into two groups and sit face to face.




(b) Ask the students from group A to the students of group B to stand in proper place/point based on the coordinate. If he or she is failed to stand at the exact point, he or she will be out from the game.

(c) Ask the students from group B to the students of group A to stand in proper place/point of the coordinate. If he or she is failed to stand at the exact point, he or she will be out from the game.

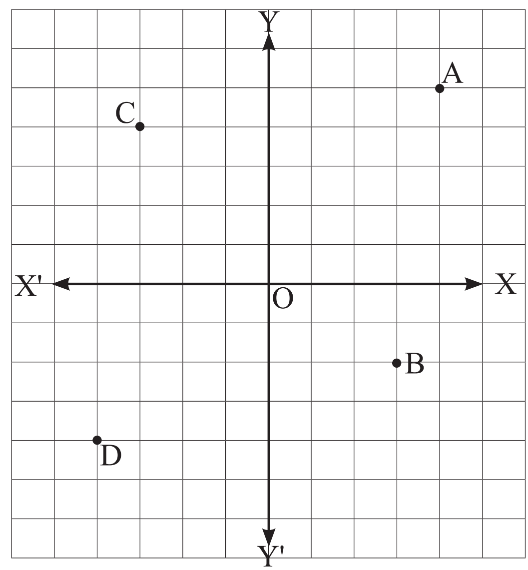
(d) Provide this opportunity to each students of both groups.

(e) At last the group that has the most students standing at the right point according to the coordinates wins.

## 17.1 Point at co-ordinates axis

### Activity 1

Make a group of suitable number of students. Study the figure in the graph and answer the following questions.




(a) What is the name of the line XOX' ?

(b) What is the name of the line YOY' ?

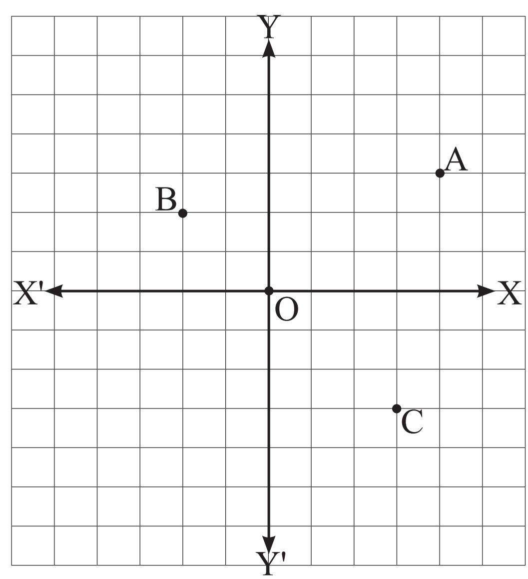
(c) How many units right and how many units up should be gone from the point O to reach the point 'A'?

(d) How many units left and how many units down should be gone from the point O to reach at the point D?

(e) What are the coordinates of O, B, C and D?

Discuss the above questions and present in the class.

### Activity 2




Be seated in group, discuss the figure and find the answer of the following questions and present in the class.

(a) In which quadrant does A lie?

(b) What is the *x*-coordinate of point A?

(c) What is the $y$-coordinate of point A?

(d) What is the coordinate of A?

(e) What is the coordinate of B?

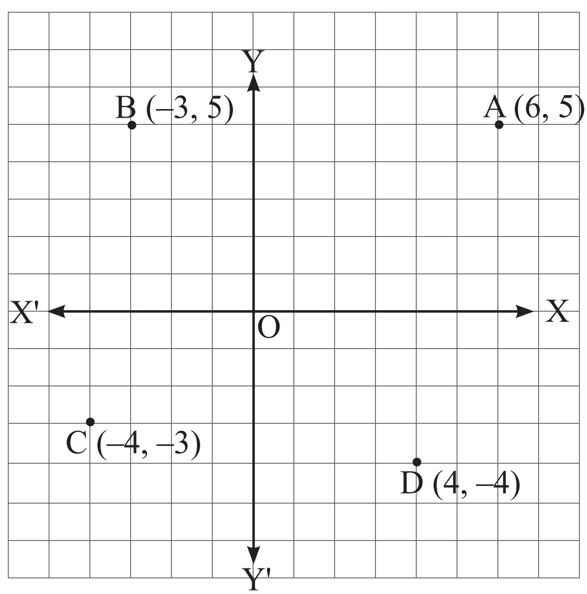
(f) What is the coordinate of C?

*While representing the point in coordinate, the x-coordinate indicates how many units are left or right from the origin whereas the y-coordinate indicates how many units are up or down from the origin.*

## 17.2 Plotting the Given Points in the Graph

### Activity 3

Plot the points A(6, 3), B(–3, 5) C(–4, –3) and D(4, –4) in graph paper. Divide the students in a certain group. Now draw the axis and write the number in graph.




(a) How can we plot point A(6, 5) in graph? Discuss in class.

(b) It should go 6 unit right from the origin and then 5 unit up from that number.

(c) In the same way discuss how you can draw points B(–3, 5), C(–4, –3) and D(4, –4) in graph paper.

(d) What is the distance between A and B? Calculate by counting the square from the point A to B.

## Example 1

Points K(3, 4), L(–3, 4), M(3, –5) and N are the vertices of a square KLMN, then

(a) Plot all the points in graph paper.

(b) Find the coordinate of the point N.

(c) Find the distance between K and L.

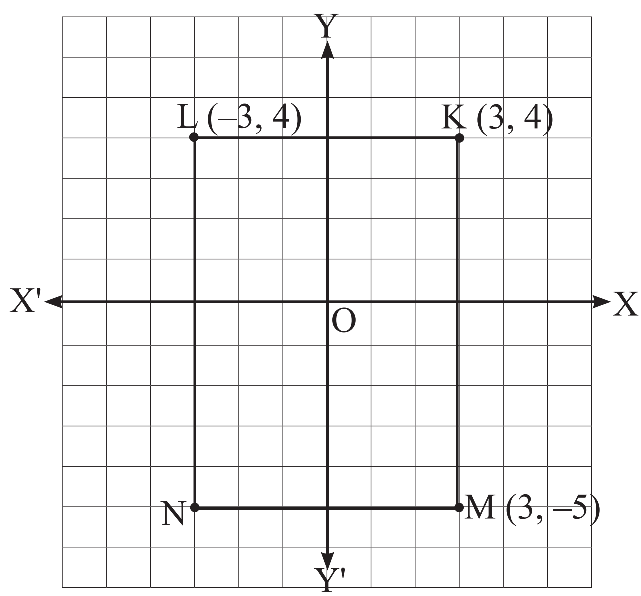
### Solution

(a) Points K(3, 4), L(–3, 4), M(3, –5) are shown in graph paper.

(b) To reach at N, it should go 3 unit left from the origin and 5 unit down from that number.

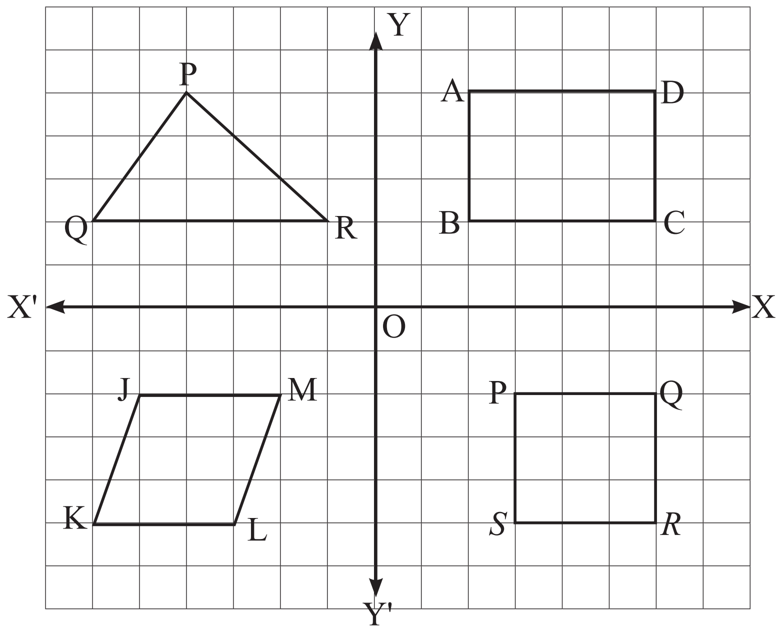
Therefore, the coordinate of N is (–3, –5).

(c) While counting the square from K and L, there are 6 squares. Therefore the distance between K and L is KL = 6 unit.



## Exercise 17

**1. Find the coordinates of the vertices of the given geometrical figures.**




2. Find the distance between the vertices of the figures in the chart.

**3. Plot the following points in graph paper.**

P(2, 2), Q(–3, 4), R(–2, 0), S(4, –4) and T(–5, –5)

**4. Plot the following points in graph paper. Join each point in order. Write the name of the shape formed.**

(a) A(4, 0), B(4, 4), C(–2, 4) and D (–2, 0)

(b) R(2, 3), S(2, –2) and T(–1, 2)

**5. Points A(–2, 3) B(2, 3) ,C(–2, 4) and D are the vertices of a rectangle ABCD.**

(a) Plot the points in graph paper.

(b)What is the length of AB?

(c)What is the length of CD?

6. **Points J(–4, 4) K(4, 4), L(4, –4) and M are the vertices of a square JKLM.**

(a) Plot the points in graph paper.

(b) What is the coordinate of M?

(c) What is the length of JK?

## Project Work

Make both X-axis and Y-axis in a big size graph paper. Make a figure in that graph. Observe the figure and write any five coordinates of vertices. Present it in the class.

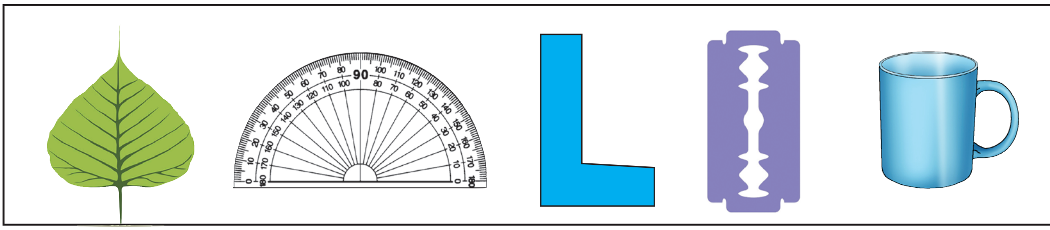
## Answer

Show the answers to your teacher.

# Lesson 18 Symmetry and Tessellation

## 18.0 Review

Observe the following figures, discuss with your friends in group and answer the questions asked.



(a) Which of the above figures can be divided into two equal parts?

(b) Which of the above figures are the symmetrical figures?

(c) Do the above figures look like the same if they are rotated about 180°?

- *The figures which can be divided into two equal parts is called symmetrical figure.*
- *The line in any figure which divides the figure into two equal part is called axis of symmetry. The axis of symmetry may be more than one in any figure.*

## 18.1 Line and Point Symmetry

### 18.1.1. Line Symmetry

**Activity 1**

Trace all the figures.

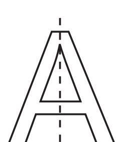


Figure (a)

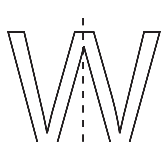


Figure (b)

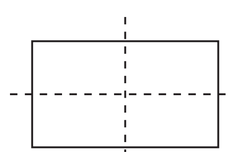


Figure (c)

Fold the figures that you have drawn that you have drawn from the dotted line (axis of symmetry).

(a) How many ways can the figure (A) be folded into two equal part?

(b) How many ways can the figure (B) be into folded into equal part?

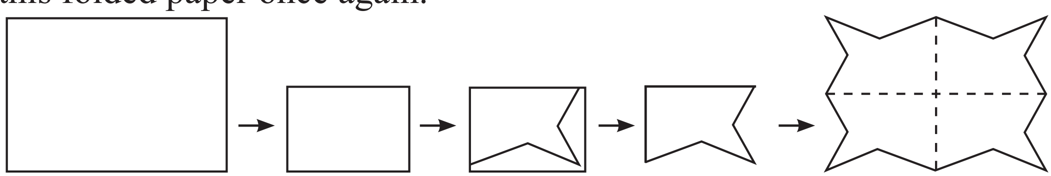
(c) How many ways can be folded into equal part?

**Discuss with your friend.**

- Figure (A) and figure (B) can be folded in one way. That's why there is one axis of line of symmetry.
- Figure (C) can be folded in two ways. That's why there is two axis of line of symmetry.

## Activity 2

Take a piece of paper and fold it from the middle as shown below. Fold this folded paper once again.



(a) (b) (c) (d) (e)

Like in figure (d) cut the outer circumference by scissors and open the folded part.

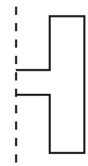
How many line of symmetry are there? Discuss in class and find it.

The dotted line which divides the figure in two equal part is called axis of symmetry. In another word, it is called mirror line.

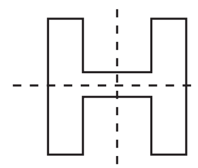
## Example 1

Axis of line of symmetry and half figure is given below. Complete it and find the number of axis of line of symmetry.

**Solution**



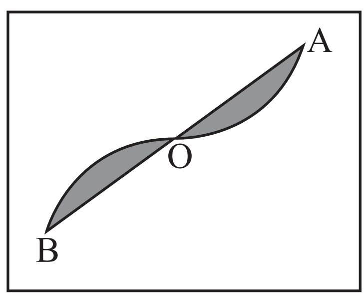
Here, axis of line of symmetry = 2



## 18.1.2 Point Symmetry

### Activity 3

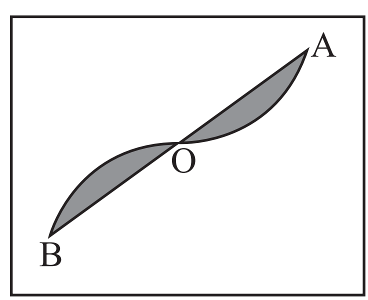
Divide all the students in a certain group and trace the given figure in transparent plastic.




In the above figure, press at O by pencil and rotate it slowly.

While rotating it,

- At what angle should the figure be rotated when it is in the equal but opposite direction from the centre?
- How many times does it overlap when it is in initial position?
- Discuss in class and present it.



## Activity 4

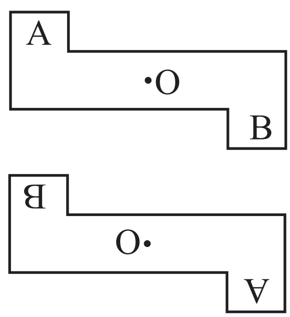
Find which letters in English alphabet remains in equal distances from center when rotating (on angle 180°) from the mid-point on the basis of point symmetry.

## Example 2

What kind of shape is made when the given figure is rotated around the centre O on the basis of point symmetry?

### Solution

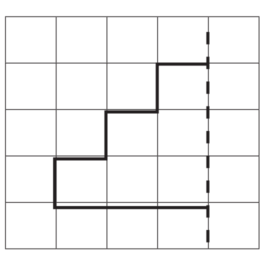
When the given figure is rotated around O (about 180°) on the basis of point symmetry, the image will be inverted (in reverse direction).



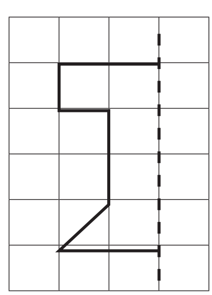

### Exercise 18.1

**1. Complete the following figures taking dotted line as an axis of reflection.**

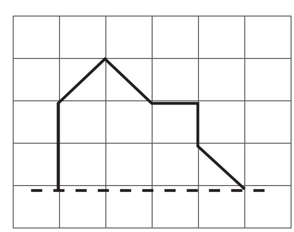
(a)



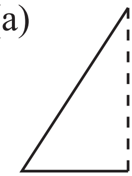
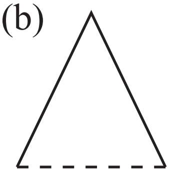
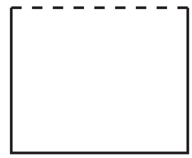
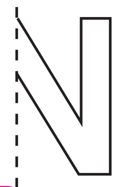
(b)



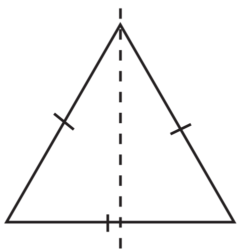
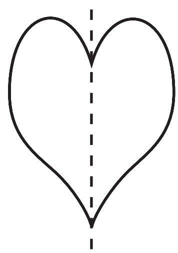
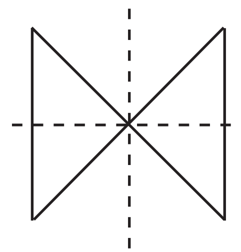
(c)

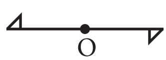


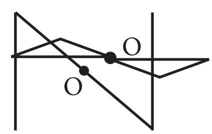
**2. In the following figure axis of reflection and half figure is given. Complete the figure and find the number of axis of line of symmetry.**

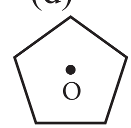
(a)  (b)  (c)  (d) 

**3. Trace each of the following point symmetry. Draw axis of line of symmetry in each. Find the number of axis of line of symmetry.**

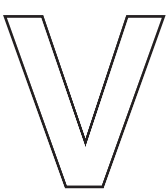
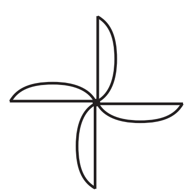
(a)  (b)  (c) 

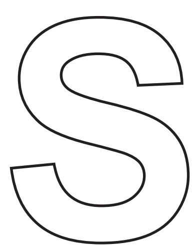
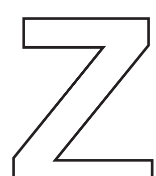
**4. Trace the following figure and rotate it taking center at O about 180°.** (a) 


(b) 


(c) (d) 


**5. Which of the following figures have point symmetry? Identify it.**

(a)  (b)  (c) 

(d)  (e)  (f) 

6. **Are the images in the given cards point symmetry or not? If yes, how ?**

## Project Work

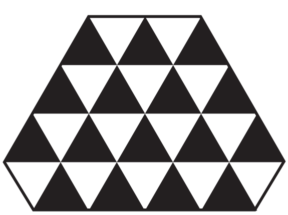
Take different colour papers. Fold it and cut it by scissors in such a way that it forms line symmetry and point symmetry. Also present these figures in class.

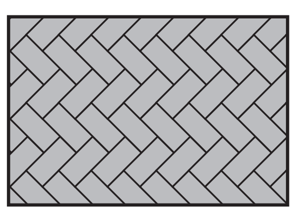
**Answer**

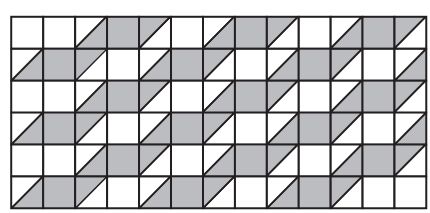
Show the answer to your teacher.

## 18.2 Tessellation

Observe the following figures. Discuss with your friends and make a list of what type and number of shapes these figure contain.







Have you ever seen this type of images in objects that are in your house like Nanglo, carpet, Doko, wall of brick or stone, football etc. ? How are these images kept? Discuss with your friends.

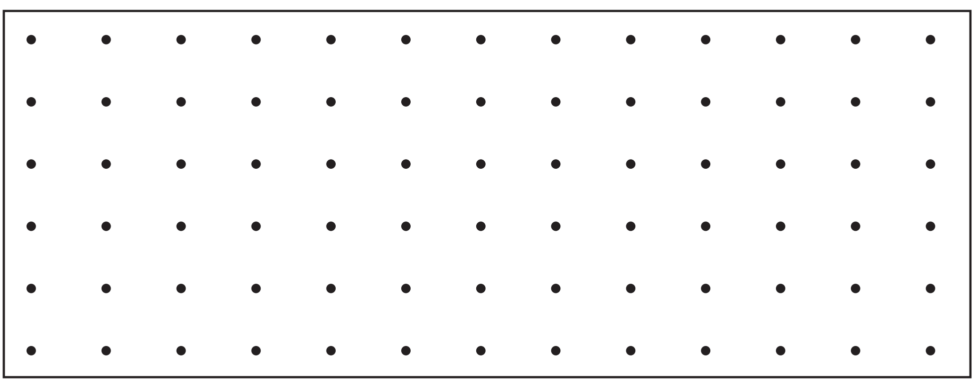
## Activity 1

All group of students cut colorful paper as shown in the fiture so as to make triangular images. Paste all the triangular pieces on a chart paper. While pasting all triangular pieces, all the surface of chart paper should be fulfilled and not be overlapped. Then discuss about tessellation with friends.

Tessellation is a Procedure of pasting one or more than one geometrical figures on a plain surface in sucha way that all the surface should be fulfilled and not be overlapped.

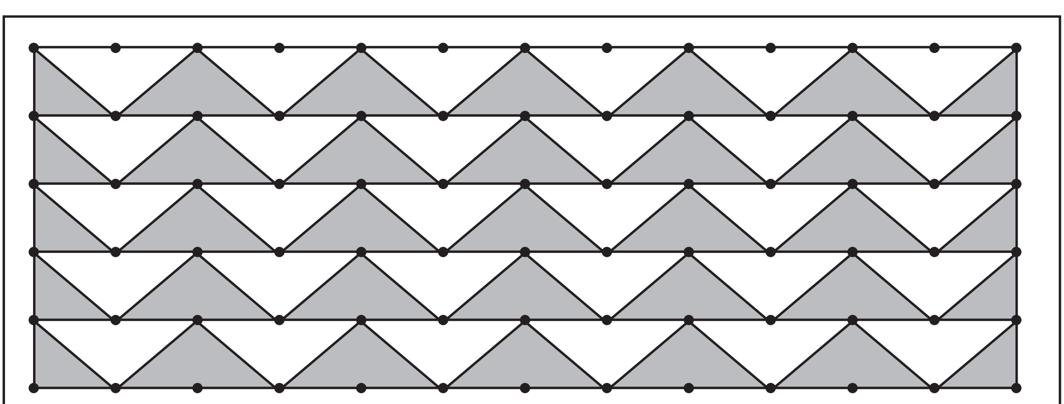
## Example 1

Make triangular tessellation by joining the given dots and color it.



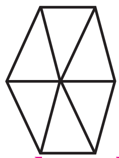
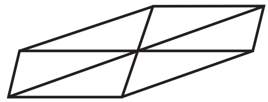
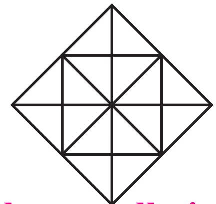
### Solution

The following triangular tessellation can be made by joining the above dots.

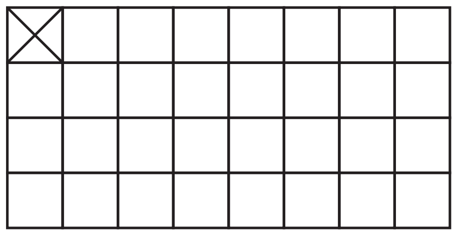


## Exercise 18.2

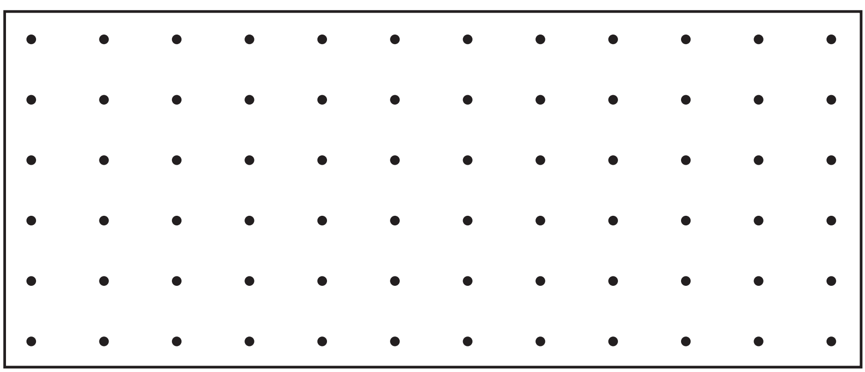
**1. Are the following figures of triangular tessellation or not? Write with reasons.**

(a) 
(b) 
(c) 

**2. Complete the following by making triangular tessellation as shown below.**



**3. Make triangular tessellation by joining the given dots and colour it.**



4. Make one - one tessellation formed from a right angle triangle and equilateral triangle.

### Project Work

Observe school, wall of house, tiles used in bathroom, carpet, football and volleyball and make triangular tessellation in your exercise. Color it and present it in the class.

### Answer

Show the answer to your teacher.

Lesson 19

# Transformation

## 19.0 Review

Observe the following figures and discuss the following questions in the class.

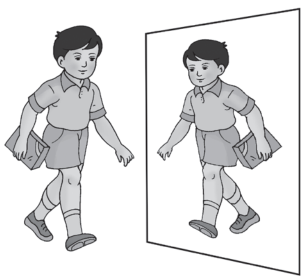


Fig. (1)

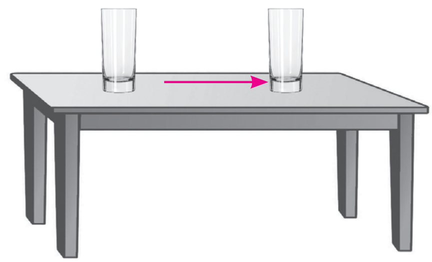


Fig. (2)

(a) In figure 1, are the man and his image in equal distance from the mirror?

(b) In figure 1, are the man and his image equal in shape and size?

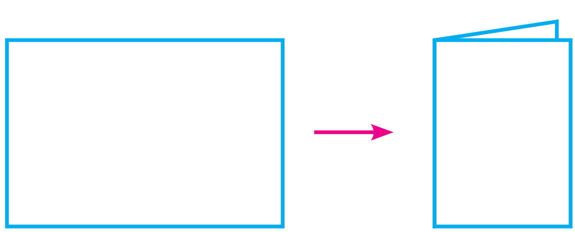
(c) In figure 2, are there any changes in shape and size while changing the position of glass from one place to another?

A transformation is the change of position and shape of any object by a certain rule.

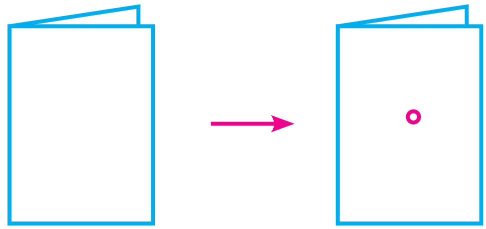
## 19.1 Reflection

### Activity 1

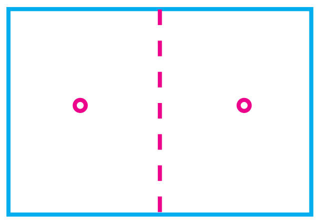
Take one - one piece of paper and fold it in two equal part as shown in the figure.



Now, make a small hole in the middle of the folded part by a compass or a pen.



Then, open the folded paper as shown in the figure below.



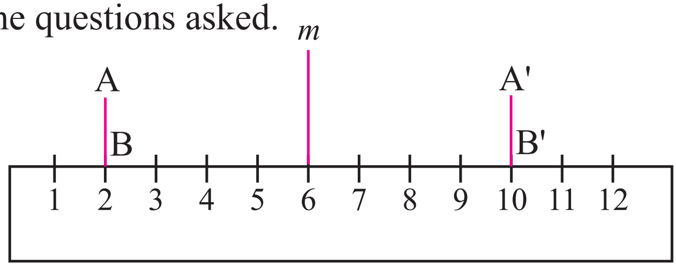
Now measure the distance between the dotted line and two holes. What conclusion did you get?

From the above discussion, the following conclusion can be made:

- Out of two holes one hole is the image of another hole (object).
- The line formed while folding the paper (dotted line) is called axis of reflection.
- The object and its image are in equal distance from the axis of reflection.

## Activity 2

Observe the following figures discuss with your friends and seek the answers to the questions asked.




(a) In the figure what is the image formed from the reflection of the line AB?

(b) Are the line AB and A'B' on equal in shape and size?

(c) Which is the axis of reflection in the figure?

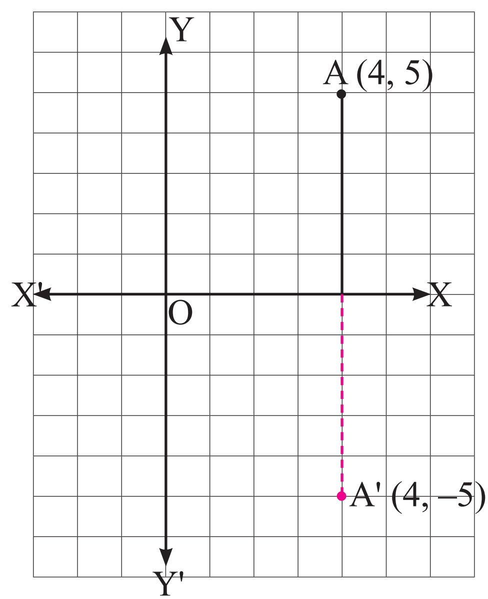
(d) Are the line AB and A'B' in equal distance from the axis of reflection?

(e) Are the line AB (object) and the line A'B'(image) congruent?

- The axis of reflection is a line from which the object is reflected.
- Image is the reflection of real object.
- The object and its image are in equal distance from the reflection line.
- The object (geometrical figure) and its image are congruent to each other.

### (a) Reflection on X- axis

Like in the figure, draw X-axis (XOX') and Y-axis (YOY') on your copy. Now reflect the point A taking X-axis (XOX') as a axis an reflection. Write A' for the image. The distance of A and A' is equal from X–axis.



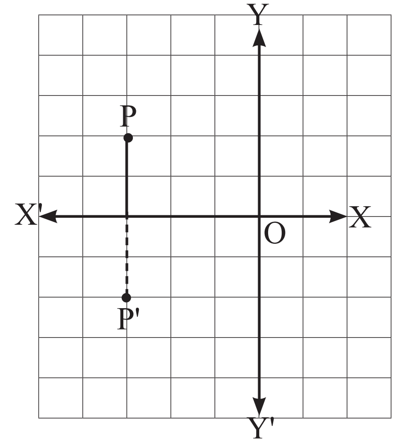

Discuss with your friend and write the co-ordinate of A and A'.

Here, in graph the coordinate of A and A' are (4, 5) and (4, –5).

The image of the point $(x, y)$ is $(x, -y)$ under the reflection X-axis. i.e. The $x$-coordinate remains the same and sign of y-coordinate will be changed.

## Example 1

**Plot the point P(–3, 2) on graph paper and find its image under reflection about X-axis.**



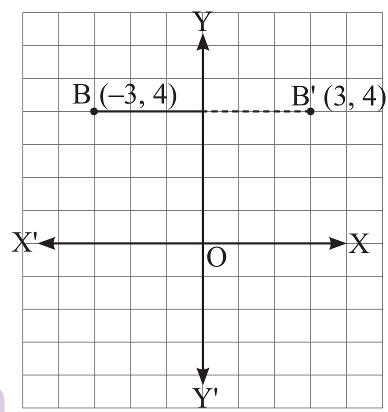

### Solution

Here P($x$, $y$) = P (–3, 2)

In graph, the image of P(–3, 2) under reflection about X-axis is (–3, –2)

Therefore P($x'$, $y'$) = P'(–3, –2)

### (b) Reflection on Y- axis



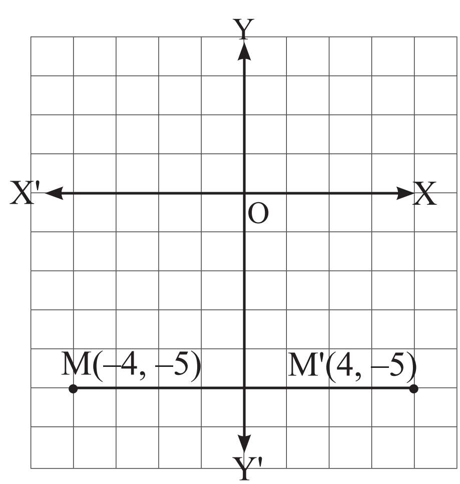

Like in graph, take a point B and reflect it about the line YOY'. The point B' is the image of B. Both B and B' are equal distance from axis of reflection.

Now count and find the co-ordinates of B and B'. The co-ordinates of B and B' are ( -3, 4) and (3, 4) respectively.

The image of the point ($x$, $y$) is (–$x$, y) under reflection about Y-axis. i.e. the $y$-coordinate remains same and sign of $x$-coordinate will be changed.

## Example 2

**Plot the point M(–4 , –5) on graph paper and find the image of point M under reflection about Y-axis.**




### Solution

Here M(x, y) = M (–4, –5)

The image of M(–4, –5) is

M'(4, –5) under reflection about Y-axis

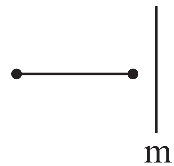
Therefore, M'(x', y') = M' (4, –5)

## Exercise 19.1

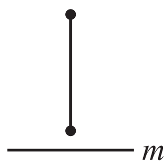
**1. Draw the image of given geometrical figure with reflection line m.**

(a)

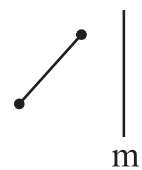

(b)




(c)

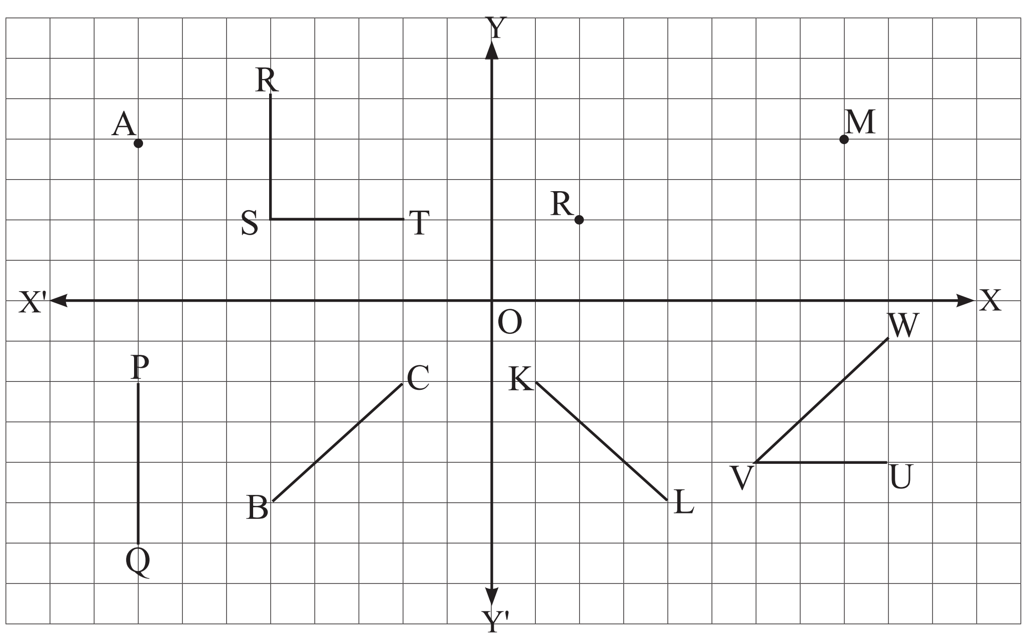



(d)




**2. Write the image of the given geometrical figures with reflection line XX' (X-axis)**

| | | |
|---|---|---|
| (a) Point A | (b) Point M | (c) Point R |
| (d) Line BC | (e) Line PQ | (f) Line KL |
| (g) ∠RST | (h) ∠UVW | |



**3. Write the image of the given geometrical figures with reflection line YY' (Y-axis).**

(a) Point F

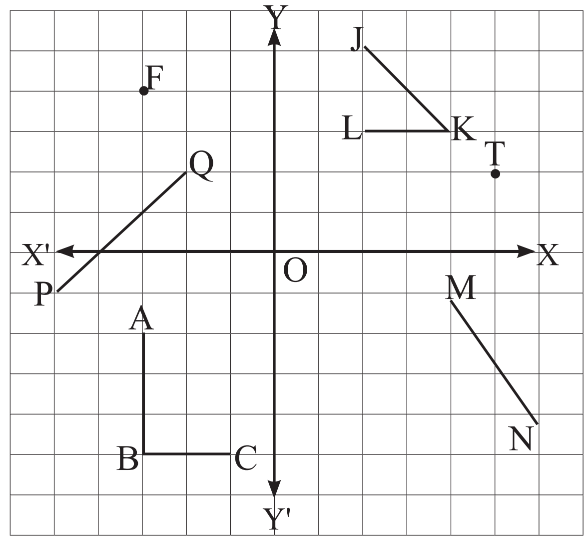
(b) Point T

(c) Line PQ

(d) Line MN

(e) ∠ABC

(f) ∠JKL




**4. Using graph paper, find the image of the following points under reflection about X-axis.**

| | | | | | |
|---|---|---|---|---|---|
| (a) | (1, 2) | (b) | (3, 2) | (c) | (–4, 4) |
| (d) | (–2, 6) | (e) | (8, –7) | (f) | (9, –10) |
| (g) | (–6, –9) | (h) | (–5, –8) | (i) | (–9, 7) |

5. Using graph paper, find the image of the points in question 4 under reflection about Y-axis.

6. Find the image of point T'(3, –4) under reflection about X-axis and find T. Also find the length of TT'.

7. Find the image of point P(3, –4) under reflection about Y-axis and find P'. Also find the length of PP'.

## Project Work

Search any five conditions of reflection that are found around your house, school, and road and present them in the class.

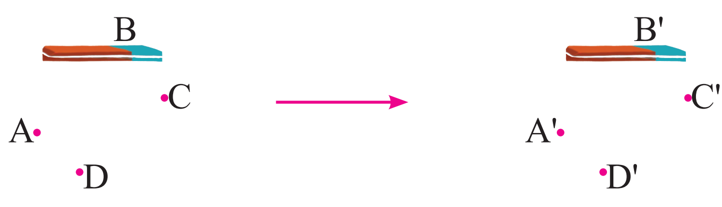
## Answer

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4. | (a) | (1, –2) | (b) | (3, –2) | (c) | (–4, –4) |
| | (d) | (–2, –6) | (e) | (8, 7) | (f) | (9, 10) |
| | (g) | (–6, 9) | (h) | (–5, 8) | (i) | (–9, –7) |
| 5. | (a) | (–1, 2) | (b) | (–3, 2) | (c) | (4, 4) |
| | (d) | (2, 6) | (e) | (–8, –7) | (f) | (–9, –10) |
| | (g) | (6, –9) | (h) | (5, –8) | (i) | (9, 7) |
| 6. | T'(3, 4), 8 unit | | 7. | P' (–4, 5), 8 unit | | |

## 19.2 Translation

### Activity 1

Draw a straight line in your exercise book. Keep one eraser at the end of line and give dot on the four corners of the eraser. Name the dots A, B, C and D.




Then drag the eraser towards another end and give dots on the four corners of the eraser. and name then A', B', C' and D '.

Now discuss with friends and find the answer of the following questions.

(a) What are the relationship among AA', BB', CC' and DD'?
(b) Is the eraser changed in a certain direction?
(c) Are the object and its image congruent?

Depending upon above activity write the definition of translation and compare with others.

- The transformation of an object with certain distance and direction is called translation.
- Direction and the distance should be mentioned in translation.

- Object and its image are congruent to each other in translation.
- While translating a point, we draw a parallel line with given direction and magnitude.

## Example 1

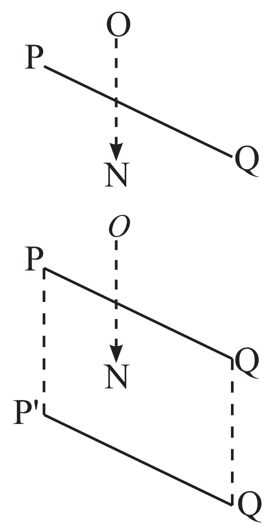
**Translate the line segment PQ to the magnitude and direction of line ON.**

### Solution

(a) From the point P, draw a line PP' with the same direction and magnitude of ON.

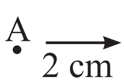
(b) From the point Q, draw a line QQ' with the same direction and magnitude of ON.

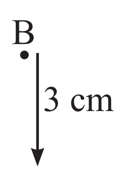
(c) Join P' and Q'. Hence, P'Q' is the image of the line PQ.

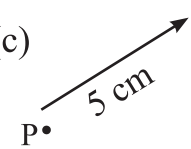


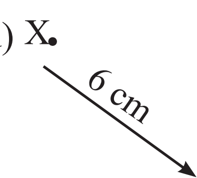

## Exercise 19.2

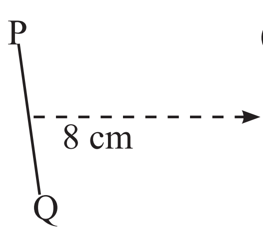
**1. Translate the given point and line segment in given magnitude and direction.**

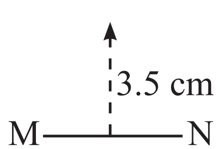
(a)


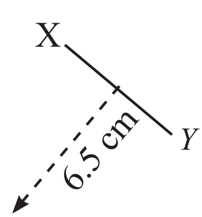

(b)



(c)


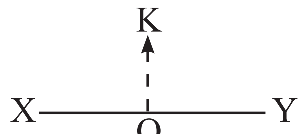

(d)



(e)


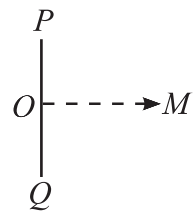

(f)



(g)


**2. Translate line XY with magnitude and direction of OK.**

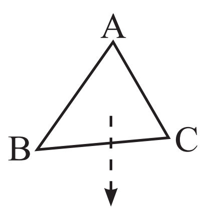



**3. Translate line PQ with magnitude and direction of OM.**

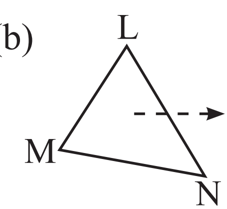



**4. Draw the image of the given geometrical figures with given magnitude and direction.**

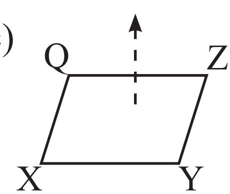
(a)




(b)




(c)




5. While playing slide a child came 4 m down. Is it translation? Give reason.

6. Do the magnitude and direction of all tip of a copy remains the same if tip of a copy is drawn 1m towards the person?

7. Do the magnitude and direction of all tip of a book which is kept on newsprint or drawing paper remains the same if tip of a book is drawn 10 cm towards the person? Is this translation?

## Project Work

Draw a straight line on a paper. Take a point A over it. Now find the image of that point translating 15 cm horizontally on the right side. Present the figure/image in the class.

### Answer

Show the answers to your teacher.

## Lesson 20 Bearing and Scale Drawing

### 20.0 Review

Study the given compass and discuss the following questions.


(a) What is this instrument used for?

(b) What does NSE and W represent?

(c) Which direction is shown by compass?

(d) In this device which direction is taken as a base direction?

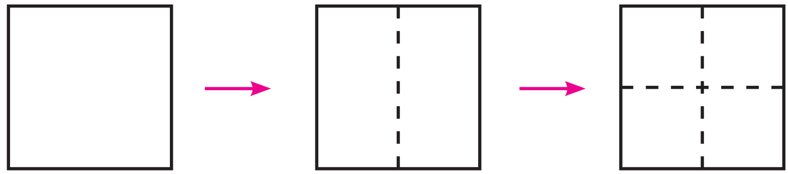
(e) What does NE, SE, SW, NW in this device?

| | |
|---|---|
| N $\Rightarrow$ North | NE $\Rightarrow$ North East |
| S $\Rightarrow$ South | SE $\Rightarrow$ South East |
| E $\Rightarrow$ East | SW $\Rightarrow$ South West |
| W $\Rightarrow$ West | NW $\Rightarrow$ North West |

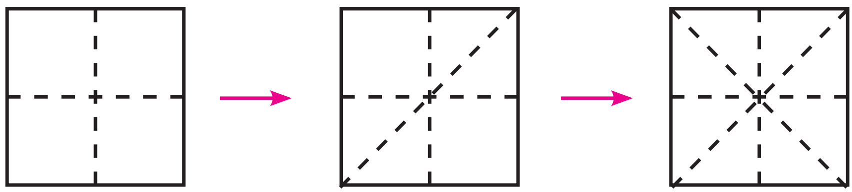
### 20.1 Bearing

**Activity 1**

Make small groups in class. Take a piece of paper by each group and fold it two times as shown in the figure.



Fold it two times again from the corner.



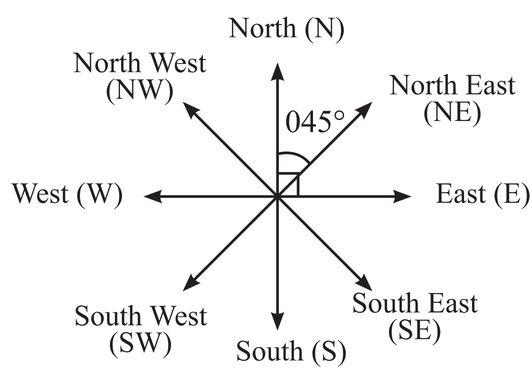
Now open the folded part. Then write the name of each folded line as shown in the figure. Discuss with your friends and find the answers of the following questions.

(a) How many directions are there in the figure (sample)? What are they?

(b) What angle does north east (NE) line make?

(c) What angle is made between north and north-east? Measure it.

(d) Are the angles between North and west, west and south, south and east also 90°?




(e) Are the angle between north and north-west, west and south-west, east and south east 45° each?

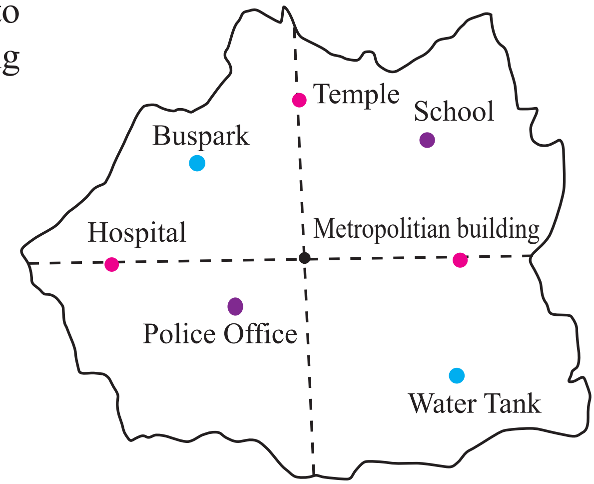
The way of presenting the position between to places in the clockwise direction as a three-digit angle on the basis of line that represents the north direction is called bearing.

## 20.1.1 Map Reading

### Activity 2

Some places of a municipality are given in the figure below. Trace this figure in your exercise book. Take the municipality building as a base and discuss with the friends to find the bearing of the following places and present in the class.

(a) School
(b) Temple
(c) Bus park
(d) Police station
(e) Hospital
(f) Water tank

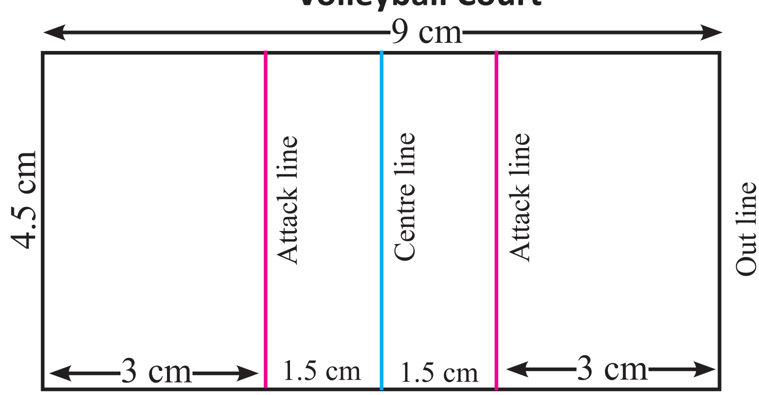



## 20.2 Scale Drawing

### Activity 3

Be seated in an appropriate group. Study the given drawing and discuss the following questions.

**Volleyball Court**




Scale: 1:200 (1 cm = 200 cm)

| S.No. | Name of the line | Measurement of line | Measurement of actual court | Ratio of the measurement of actual court and map | Conclusion |
|---|---|---|---|---|---|
| 1. | Length of court | 9 cm | 18 m | 1 : 200 | |
| 2. | Breadth of court | | | | |
| 3. | Distance between attack line and centre line | | | | |

- What is the ratio between breadth of court and breadth of map?
- What is the ratio between length of court and length of map?
- If the distance between centre line and attack line is 6 cm,what is the real distance?
- Prepare a map of above volleyball court taking scale 1cm = 1 inch in drawing paper.
- What conclusion can be drawn from the above discussion?

1. We can draw the figure of very small and very big object by taking certain scale.
2. We make a certain scale by taking the real object and figure in a small or big scale as required.
3. We can take real measurement by using the scale.

## Example 1

A map is prepared using scale 1 cm = 500 m. If the distance between two places is 9 cm in map, what is the real distance between them?

## Solution

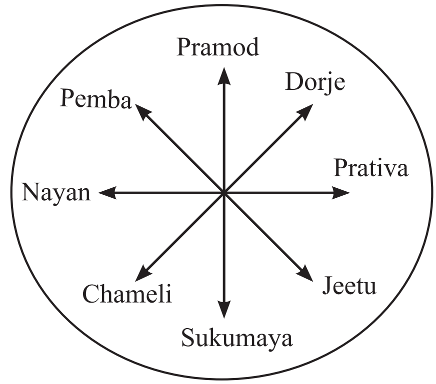
Here,

scale 1 cm = 500 m real distance.

Scale 9 cm = $(9 \times 500)$ m = 4500 m

## Exercise 20

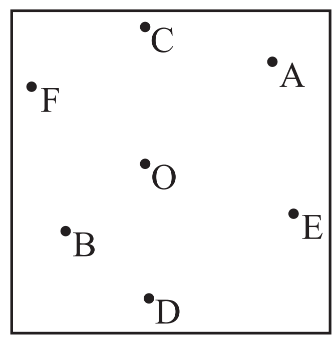
**1. All the students of a class are standing in the ground as shown in the figure. Now, answer the following questions.**




(a) What is the bearing of the place Dorje is standing?

(b) What is the bearing of the place of Jakir?

**2. Answer the following questions based on the given figure.**




(a) Find the bearing of A from the point O.

(b) Find the bearing of C from the point O.

(c) Find the bearing of F from the point O.

(d) Find the bearing of B from the point O.

**3. Study the real map of Nepal and answer the following questions.**

(a) Write the bearing of Nepalgunj from Pokhara.

(b) Write the bearing of Manang from Kathmandu.

(c) Write the bearing of Taplejung from Janakpur.

**4. Find the real distance between two places.**

(a) The distance between two place in map = 9 cm (scale 1cm = 200 m)

(b) The distance between two place in map = 3.5 cm (scale 1cm = 500 ft).

5. The real length of a ground is 125 m and breadth 75 m. If 1 cm = 10 cm, draw the figure of ground.

6. 1cm represents 2 feet. Draw a map of a room whose length is 20 ft and breadth is 18 ft.

7. The given figure of an electric pole is in 1:100 cm scale.Find the real height of that pole.

8. Let all the friends stands in the school playground as shown in question 1. Find the bearing of each friend standing in all direction. Also call each friend with bearing name.

## Project Work

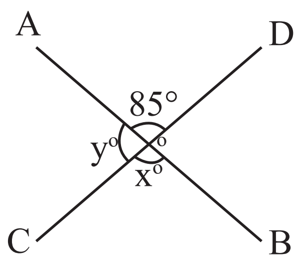
1. Take a map of Nepal and confider any one province headquarter as the centre. Now, write the names of places that are in eight directions (N, S, E, W, SE, SW, NW, NE).
2. Goverment has planned to settle people who were victimized from land slide in safe places. The government decided your 5-member expert group to manage it and provided your group a map of open land. How can you settle them? Make a plan with figure and suggest the government.

## Answer

Show the answers to your teacher.

## Miscellaneous Exercise

**1. Find the value of *x* and *y* based on the given figure.**




(a) Which angles in the figure are congruent?

(b) What can you notice in the figure point symmetry or line symmetry?

**2. Plot all the following points in graph paper. Join the points one by one. Write the name of the figure formed.**

A(–4, 2)   B(4, 3)   C(2, 5)

(a) Write the coordinate of A'B'C' after reflection of ABC about X-axis.

(b) Why are ABC and A'B'C' congruent figures?

(c) Is any rule applied when ABC is reflected into A'B'C'? Is it possible to derive any formula from it? Discuss with your teacher.

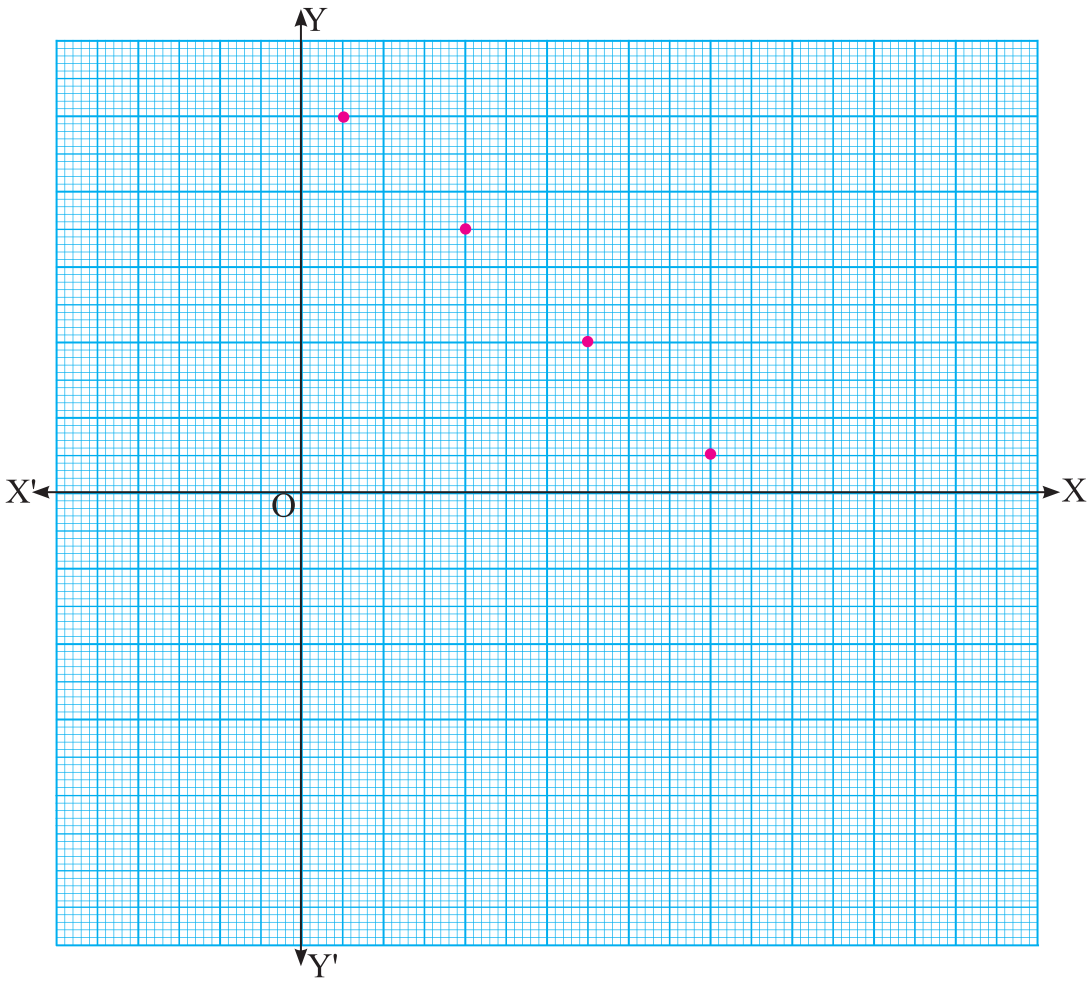
3. What is the length of diagonal of a rectangle whose length and breadth are 4 cm and 3 cm respectively? Is the diagonal of the rectangle axis of line symmetry ? Give reason.

4. If 1 cm represents the real length of 10 m, then draw the figure of a football ground having 90 m length and 45 m breadth. What kind of angles are formed at the four corners of this type of figure? Taking diagonal as a axis of reflection, what kind of figure is made? Discuss with friends.

5. English alphabet V has one axis of line symmetry and H has two axis of line symmetry. Find other letters in English alphabet which have axis of line symmetry. List out them.

**6. Write the coordinate of the following points.**

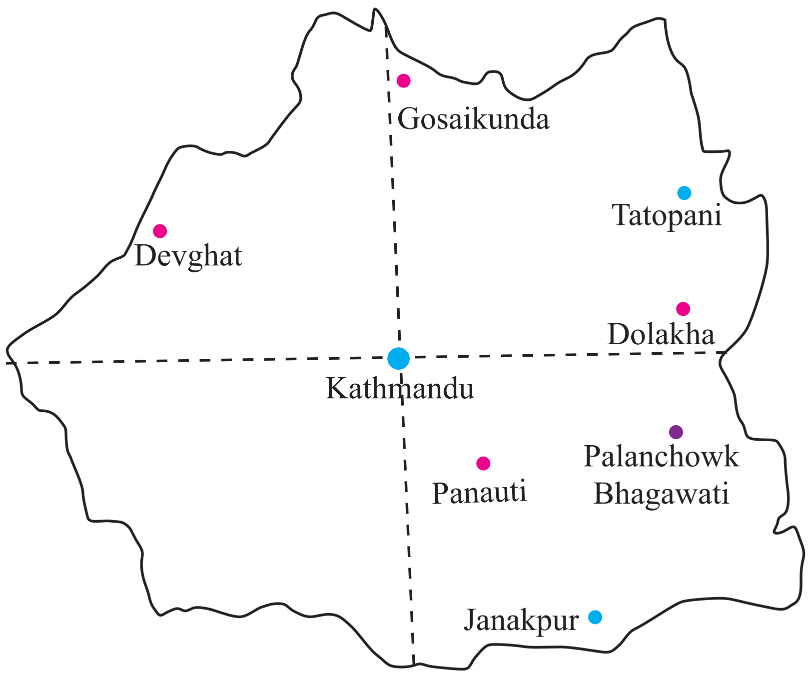
Find the image of all the points after reflection on X-axis and Y-axis.



**7. A vehical is at S which is 200km east from the point T. If it travelled 300km west from S to the point U. Now find the following by scale drawing.**

(a) Distance from T and U (b) Bearing of T from U.

**8. Some places are indicated in the given figure. Trace the figure in your exercise book. Find the bearing of the following places by taking the base as Kathmandu.**




(a) Gosainkund

(b) Tatopani

(c) Devghat

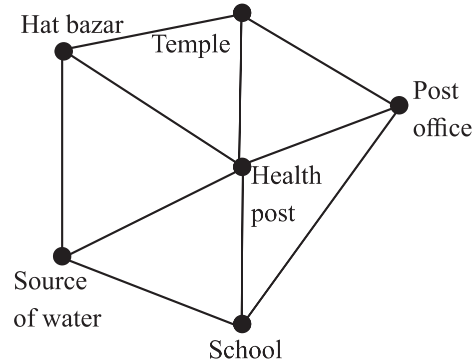
(d) Dolakha

(e) Palanchok Bhagawati

(f) Panauti

(g) Janakpur

**9. Some main places of a village are given in the figure. If the scale is given 1 cm = 100 m, use scale find the real distance of the following from the Health post.**




(a) Temple

(b) Source of water

(c) School

(d) Hat bazar

(e) Post office

(f) Write the bearing of school, Hatbazar and post office from the health post.

(g) What is the bearing of temple and school from the health post?

**Answer**

Show answers to your teacher.

# Lesson 21 Presentation of Data

## 21.0 Review

The numerical sales details of books in a stationery in one month are given below.

| | | | | | |
|---|---|---|---|---|---|
| 13, | 14, | 13, | 16, | 18, | 20 |
| 13, | 25, | 10, | 18, | 12, | 10 |
| 28, | 25, | 12, | 15, | 17, | 15 |
| 25, | 24, | 20, | 10, | 25, | 18 |
| 20, | 22, | 18, | 15, | 13, | 20 |

Complete the above information in the following table.

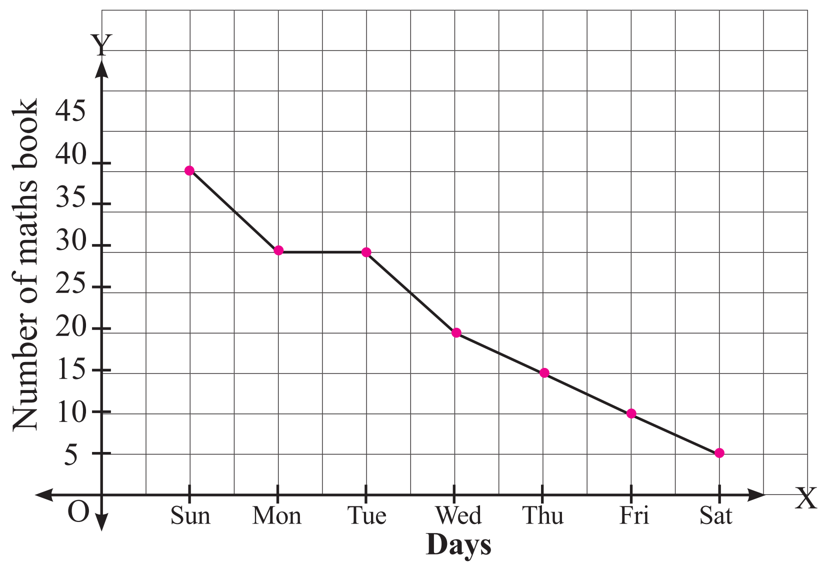
| No.of book | Tally bar | frequency | Cumulative frequency |
|---|---|---|---|
| 10 | III | 3 | 3 |
| 12 | II | 2 | 3 + 2 = 5 |

What is the name of the above table? Discuss with your friends.

## 21.1 Line Graph

### Activity 1

Dolma presented the one week sales of mathematics book by a stationery store in line graph as shown below. Study the given line graph and answer the following question.




(a) How many mathematics book were sold on Monday?

(b) How many mathematics book were sold on Friday?

(c) On which days were equal number of books sold?

(d) On which day was maximum number of books sold?

Line graph is a line joining the variable price and its frequency on graph paper

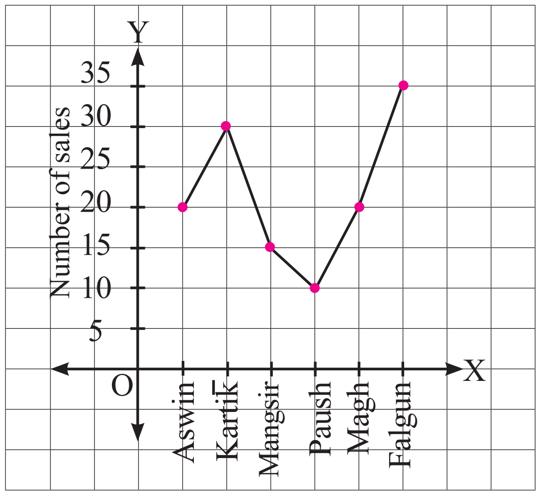
**Steps for construction of line graph**

1. Write the data in a table.
2. Write frequency in Y-axis and quantity in X-axis.
3. Select the proper scale.
4. Plot the points on graph and join by line.

## Example 1

The sales of mobile in a shop in six months is shown in the following line graph. Study the given line graph and answer the following questions.

**Sales details of Mobiles**




(a) In which month was the maximum number of mobiles sold?

(b) In which month was the least number of mobiles sold?

(c) How many mobiles were sold in Aswin?

(d) Make a frequency table on the basis of this line graph.

**Solution**

The answers of above questions are as follows.

(a) The maximum number of mobiles (35) was sold in Falgun.

(b) The least number of mobiles (10) was sold in Push.

(c) 20 mobiles were sold in Ashoj.

| Month | Aswin | Kartik | Mangshir | Paush | Magh | Falgun |
|---|---|---|---|---|---|---|
| Number of Mobiles | 20 | 30 | 15 | 10 | 20 | 35 |

## Example 2

The measurement of the quantity of rainfall of a particular place in six days in a week is given below.

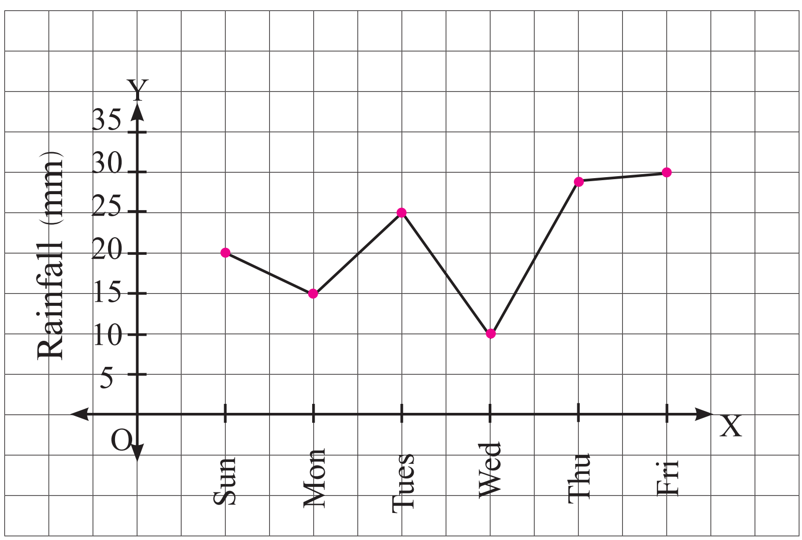
| Days | Sunday | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|---|
| Rainfall | 20 mm | 15 mm | 25 mm | 10 mm | 28 mm | 30 mm |

Show the above information in line graph.

### Solution

Here, write the days in X-axis and the quantity of the rainfall in Y-axis.

**Rainfall of 6 days**




Days

## Exercise 21.1

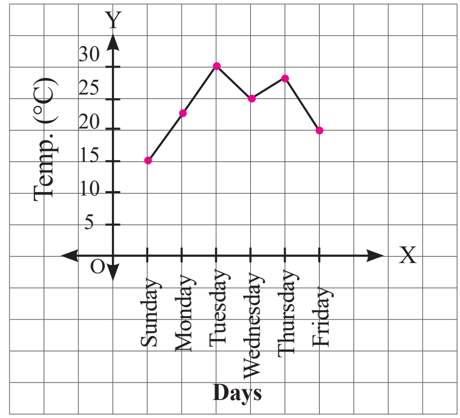
**1. The temperature of Kathmandu in 6 days is given in line graph. Study it and answer of the following.**

(a) Which day has the highest temperature?

(b) Which day has the lowest temperature?

(c) What is the temperature on Wednesday?

(d) Show the given line graph in frequency table.




**2. The enrollment in grade-7 of Sarada Basic School of in 6 years are as follows:**

| Year | 2074 | 2075 | 2076 | 2077 | 2078 |
|---|---|---|---|---|---|
| Enrollment | 20 | 26 | 18 | 14 | 12 |

Show the information in line graph.

**3. The response of class 7 for the questions of preventive measures from covid-19 are as follows.**

| Means of protection | Mask | Sanitizer | Faceshield | Gloves | Soap |
|---|---|---|---|---|---|
| Number of students | 30 | 22 | 18 | 8 | 25 |

**4. Show the following table in line graph.**

(a)

| Class | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of Students | 26 | 24 | 28 | 40 | 35 | 45 | 50 |

(b)

| Quantity (kg) | 2 | 4 | 6 | 9 | 12 |
|---|---|---|---|---|---|
| Price (Rs.) | 500 | 1000 | 1500 | 2250 | 3000 |

5. Measure the temperature of a day in school from 10 am to 4 pm at the difference of 2 hours by thermometer and show it in line graph.

**Project Work**

Note down the temperature of a week on exactly 8 am perday from the means of radio, TV or newspaper and present it in line graph.

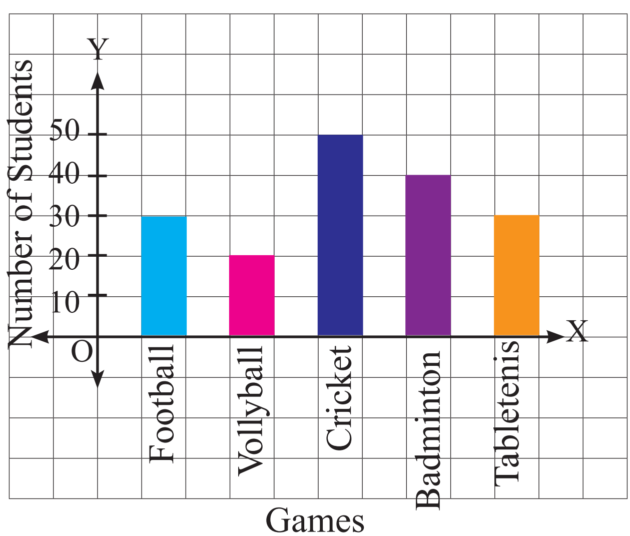
**Answer**

Show answers to your teacher.

## 21.2 Multiple Bar Graph

The following is the bar graph prepared in accordance with the students' response on the question for their choices of the games. Study it and answer the following questions.

**Games preffered by students**

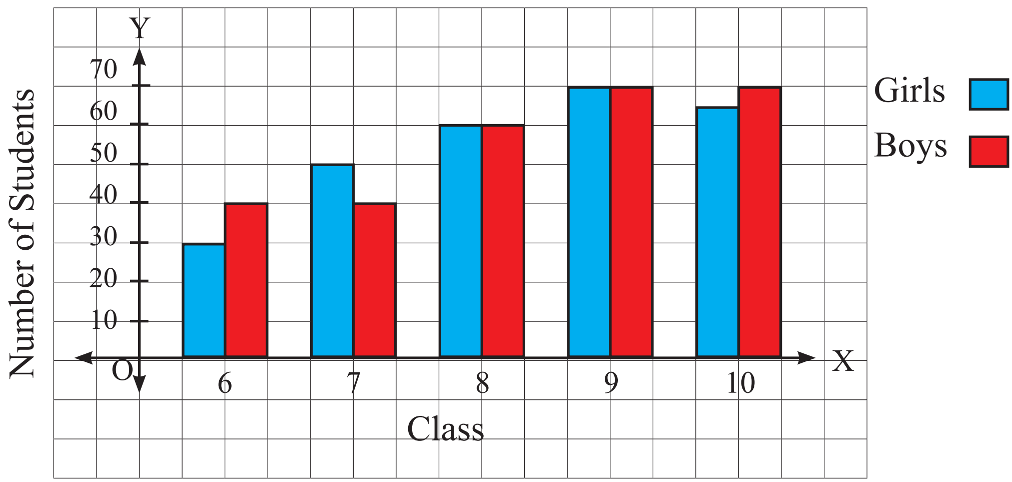



(a) What type of bar graph is this?
(b) Which is the game that is preferred by maximum number of students?
(c) Which is the game that is preferred by minimum number of students?
(d) How many students like cricket?

## Activity 1

The number of boys and girls of Saraswati Secondary School is shown as below. Discuss with friend and answer the following questions.

**Students at Saraswati Secondary School**




(a) Which class has the maximum and minimum number of students?

(b) In which class has the maximum number of girls?

(c) In which class has more boys than girls?

(d) In which class has equal number of boys and girls?

(e) What type of bar graph in this?

- *The representation of more than one interrelated information or data in bar diagram is called multiple bar diagram.*
- *The width of each bar in multiple bar graph equal like as in simple bar graph.*
- *The height of bar diagram represents number.*

## Example 1

The data of testing of ear, eye, throat and teeth in a health camp, organized by Nagar Sudhar Samiti is as follows.

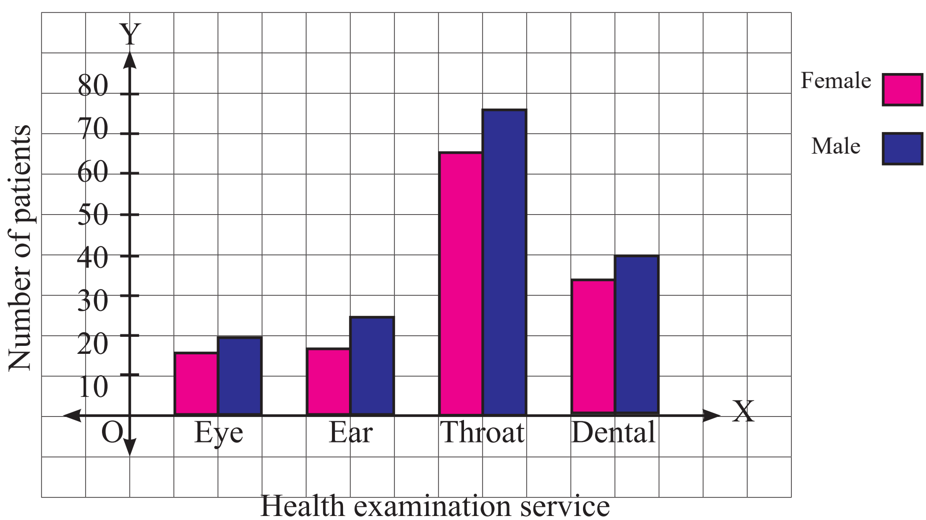
| Health examination service | Eyes | | Ears | | Throat | | Dental | |
|---|---|---|---|---|---|---|---|---|
| Tests | Female | Male | Female | Male | Female | Male | Female | Male |
| Numbers | 15 | 20 | 17 | 22 | 65 | 73 | 32 | 40 |

Show the above data in multiple bar graph.

## Solution

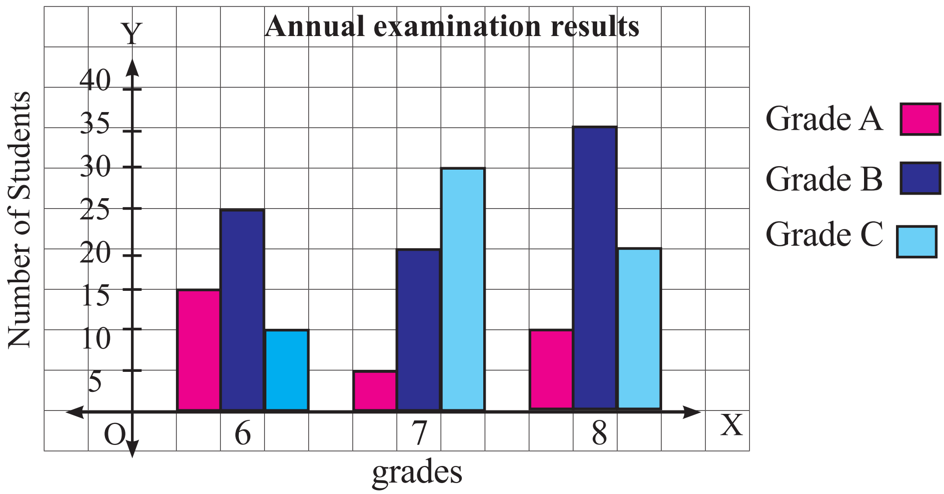
Here, taking a small box =10 person

**Health camp registration data**



## Example 2

The annual examination result of students of a school from class 6 to 8 is presented in the multiple bar diagram below. Answer the questions based on the bar graph.




(a) How many students of grade 8 got grade A?

(b) How many students from grade 6 to 8 got grade B?

(c) In which grade did the students get maximum number of grade B?

(d) In which grade did the students get minimum number of grade C?

## Solution

Here, the answers of above questions are as follows:

(a) 10 students got grade A in grade 8.

(b) The total number of students who got grade B from grade 6 to 8 = 25 + 30 + 35 = 90.

(c) The maximum number of student in grade 7 got grade A.

(d) The minimum number of student in grade 6 got grade C.

## Exercise 21.2

**1. Represent the following data in multiple bar graph.**

(a)

| Days | Sun | | Mon | | Tue | | Wed | | Thu | | Fri | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sold quantity (in Kg.) | Fish | Meat | Fish | Meat | Fish | Meat | Fish | Meat | Fish | Meat | Fish | Meat |
| | 10 | 18 | 15 | 12 | 17 | 20 | 19 | 22 | 20 | 25 | 27 | 20 |

(b)

| School | A | | B | | C | | D | | E | |
|---|---|---|---|---|---|---|---|---|---|---|
| No.of Students | Boys | Girls | Boys | Girls | Boys | Girls | Boys | Girls | Boys | Girls |
| | 250 | 210 | 125 | 175 | 310 | 350 | 425 | 400 | 520 | 550 |

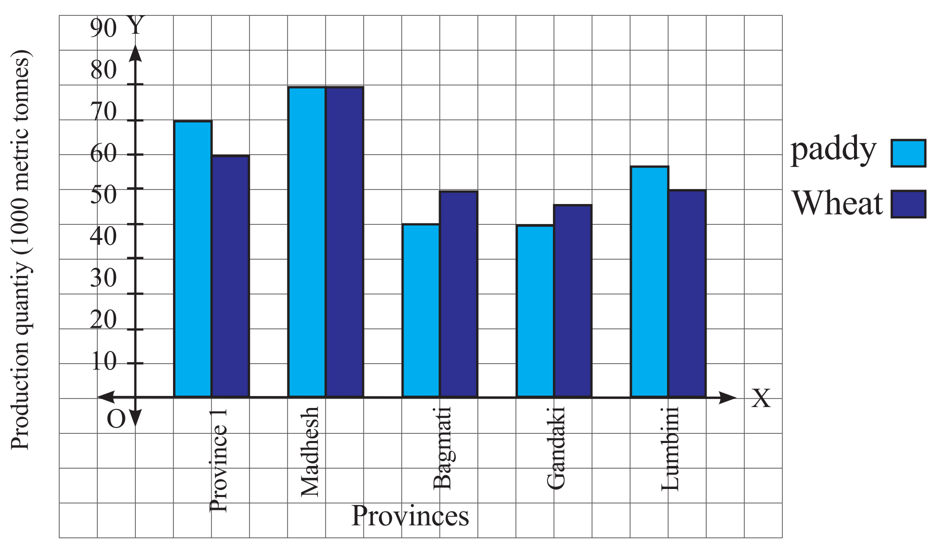
**2. The number of students who uses vehicle or walks on foot for school is as follows.**

| Means of transportation | Foot | | Bus | | Motorcycle | | Cycle | |
|---|---|---|---|---|---|---|---|---|
| Number of Students | Boys | Girls | Boys | Girls | Boys | Girls | Boys | Girls |
| | 25 | 30 | 40 | 45 | 20 | 15 | 55 | 60 |

Show the above information in multiple bar graph.

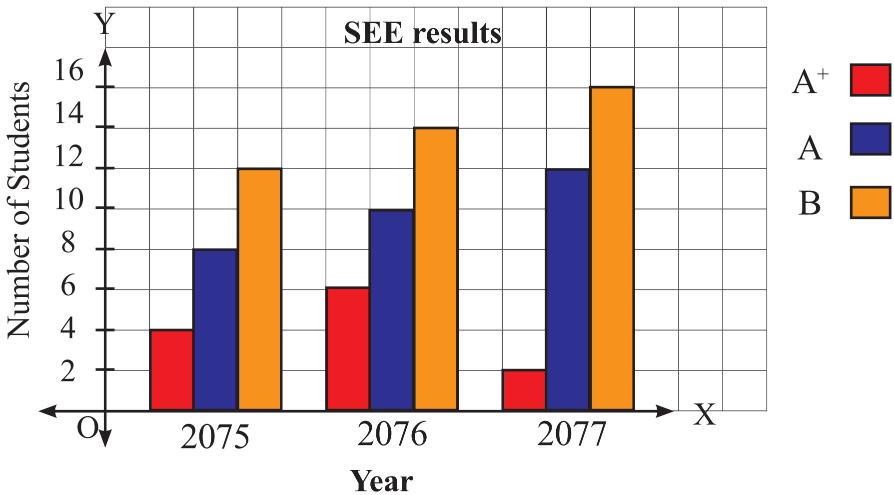
**3. The following multiple bar graph shows the quantity of paddy and wheat produced in five provinces (in 1000 metric tons). Observe the multiple bar graph and answer the questions.**

Production quantiy of paddy and wheat in five provinces



(a) Which province has the highest production of rice?
(b) Which province has the highest production of wheat?
(c) Which province has produced equal quantity of paddy and wheat?
(d) Which province has the least production of paddy?

**4. The following multiple bar graph represent the SEE result of past three years of a school. Answer the following questions based on the given bar graph.**




(a) How many students got A+ grade in 2075 BS?
(b) How many students got A grade in 2076 BS?
(c) In which year did the maximum number of student get A+ grade?
(d) In which year did the minimum number of student get B grade?

## Project Work

From an annual programme of your school, collect the data of student participation in different games like musical chair,100 m running, 200 m running, spoon race etc. and represent it in multiple bar graph.

## Answer

Show the answers to your teacher.

## Miscellaneous Exercise

**1. The maximum temperature of a city in a week of Baishakh is given in the table.**

(a) Make a bar graph based on the given data.

| Date | Maximum temp.(°C) |
|---|---|
| 16 | 35.5 |
| 17 | 37 |
| 18 | 36 |
| 19 | 32 |
| 20 | 30 |
| 21 | 33 |
| 22 | 34.5 |

(b) Represent the data in a line graph and answer the following questions.

(i) Which is the hottest day?

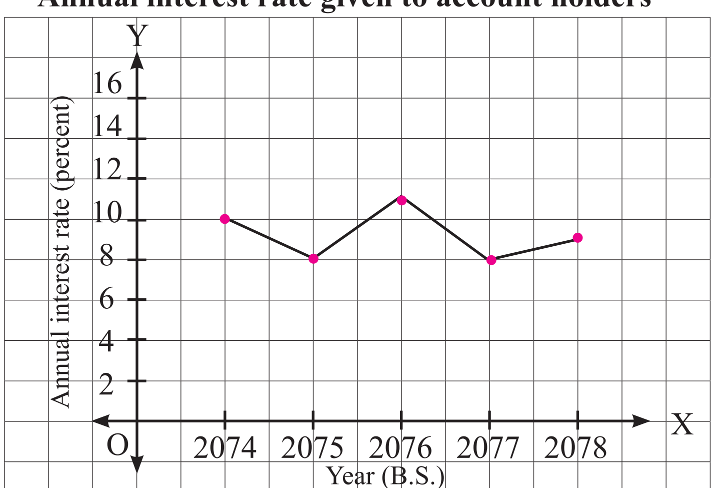
(ii) What is the maximum temperature on 20?

(iii) How much temperature is changed in a week?

(iv) What is the difference between maximum and minimum temperature?

**2. The interest rate provided to the account holders by a bank in fixed deposit is presented in the line graph below. Answer the given questions based on the line graph.**

**Annual interest rate given to account holders**



(a) Which year has the maximum interest rate?
(b) Which year has the minimum interest rate?
(c) Which year has an equal number of interest rate?
(d) Prepare a bar graph based on the data.

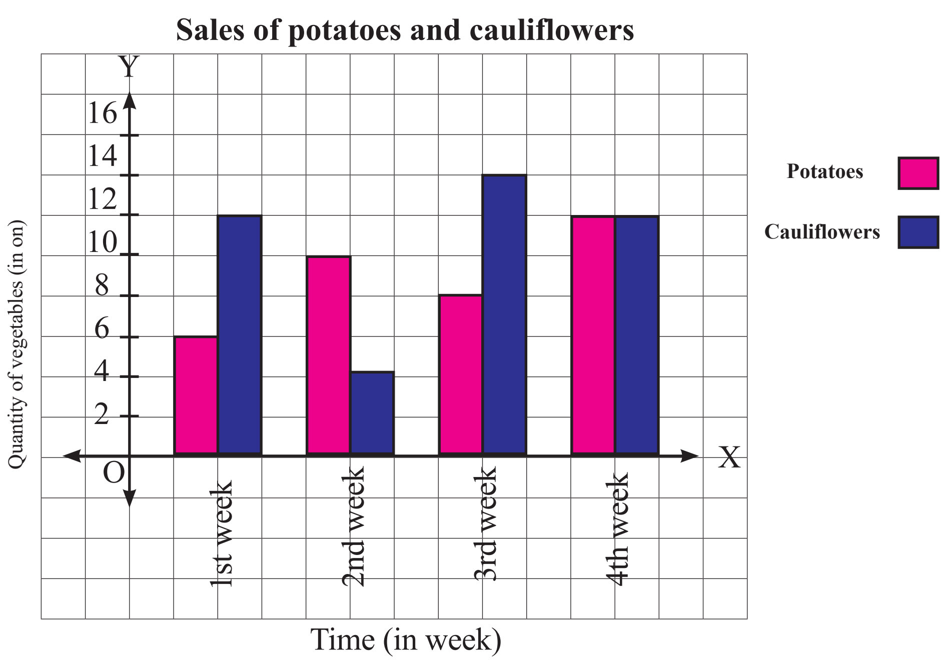
**3. The population of a village in 5 years are given below.**

| Year | 2073 | | 2074 | | 2075 | | 2076 | | 2077 | |
|---|---|---|---|---|---|---|---|---|---|---|
| Population | Female | Male | Female | Male | Female | Male | Female | Male | Female | Male |
| | 12 | 11 | 13 | 14 | 14 | 15 | 17 | 13 | 17 | 11 |

Answer the following questions based on the above data.

(a) Mention the year on which the population of male is more than female.
(b) What percentage of the population of female in more than that of male in five years?
(c) In which years are the population of male and female equal?

**4. The sales of vegetable from a shop in a week is represented in the following multiple bar graph. Answer the following questions based on the multiple bar graph.**



(a) How many kilograms of potatoes were sold in first week?

(b) In which week were potatoes and cauliflowers sold in equal quantities?

(c) In which week was the cauliflower sold the most?

(d) In which week was the potatoes sold the least?

(e) How many kilogram of potatoes and cauliflower were sold in the fourth week?

(f) What percentage of cauliflowers was sold more in the third week than in the second week?

(g) In which vegetable has maximum fluctuate (up and down) in selling price?

**Answer**

Show the answers to your teacher.